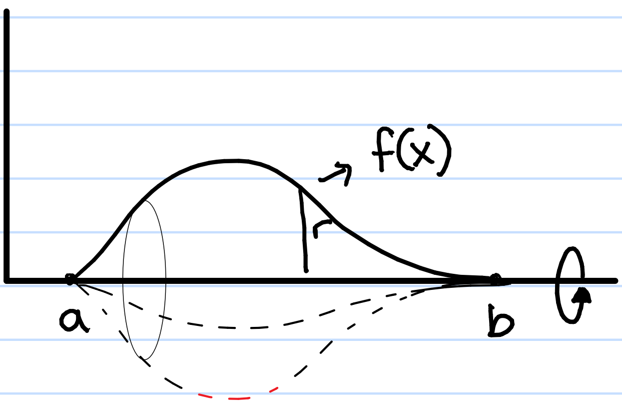
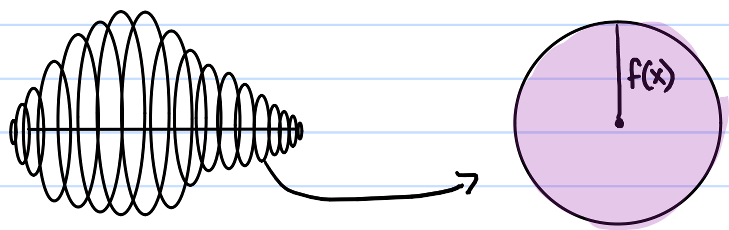
Learning Goal 7.3 Study Guide

The goal: Setup and evaluate integrals representing the volume of 3D Solids

Revolving Disks Around the x-axis



Think about a bunch of circles stacked on top of each other.

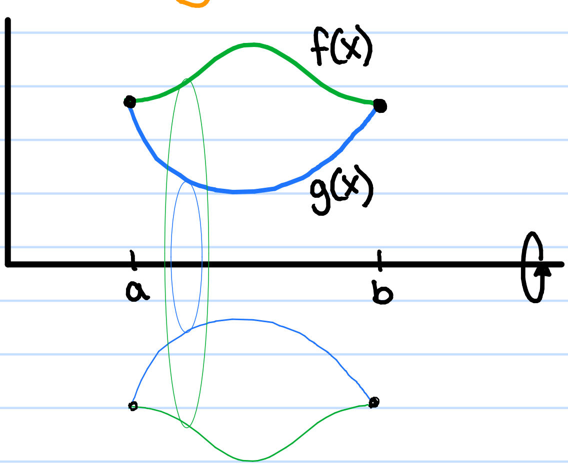


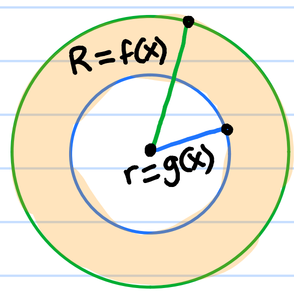
Each circle has a radius of f(x), so we need to think about adding up lots of circle areas:

$\pi r_1^2 + \pi r_2^2 + \pi r_3^2 + \pi r_4^2$ .... which we write as an integral:

$$\boxed{\int_a^b \pi (f(x))^2 \, dx} \quad \text{or} \quad \pi \int_a^b f(x)^2 \, dx$$

Revolving Washers Around the x-axis:





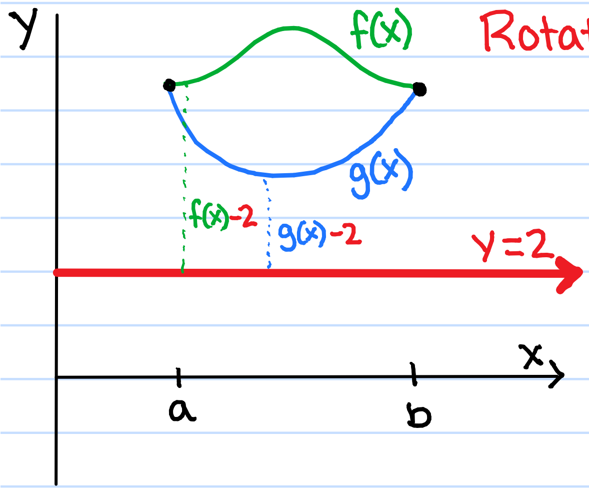
The area of the orange washer is $\pi R^2 - \pi r^2 = \pi(f(x))^2 - \pi(g(x))^2$

Each washer has area of $\pi(f(x))^2 - \pi(g(x))^2$ so the total area can be expressed as:

$$\pi \int_a^b f(x)^2 - g(x)^2 \, dx$$

where f(x) is the outer radius & g(x) is the inner radius

Rotating Around Other Horizontal Lines

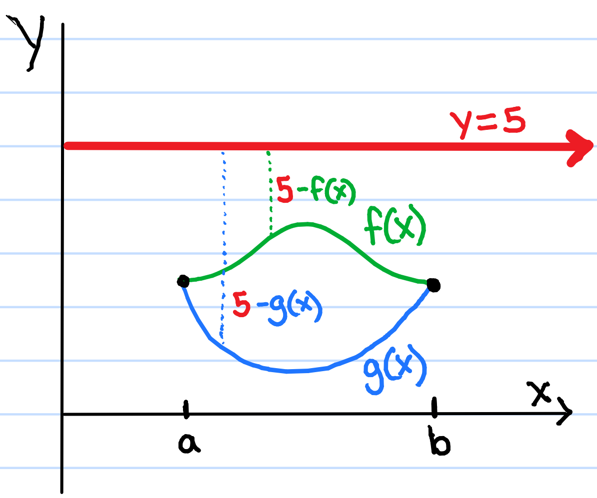


Rotating around line below region: Example $y=2$

Outer Radius $= f(x)-2$
Inner Radius $= g(x)-2$

$$\text{Volume} = \int_a^b \pi\left(f(x)-2\right)^2 - \pi\left(g(x)-2\right)^2 dx$$

Rotating around a line above the region: Example: $y=5$



Note: The outer and inner functions switch!

Outer Radius: $5-g(x)$
Inner Radius: $5-f(x)$

$$\text{Volume} = \int_a^b \pi\left(5-g(x)\right)^2 - \pi\left(5-f(x)\right)^2 dx$$