Things You Gotta Know: Rules of Exponents and Logarithms

Exponential Laws	**Logarithm Laws**
① $x^a \cdot x^b = x^{a+b}$	⑥ $\log(ab) = \log(a) + \log(b)$
② $\dfrac{x^a}{x^b} = x^{a-b}$	⑦ $\log\left(\dfrac{a}{b}\right) = \log(a) - \log(b)$
③ $(x^a)^b = x^{ab}$	⑧ $\log(a^b) = b \cdot \log(a)$
④ $x^{-a} = \dfrac{1}{x^a}$	⑨ $\log_x\left(\dfrac{1}{x^a}\right) = -a$
⑤ $x^0 = 1$	⑩ $\log_x 1 = 0$
	⑪ $\log$

Note: All rules for logarithms shown apply to the natural log → $\ln(x)$.

The natural log is a logarithm with base e.

Rule ⑦

Examples:

a) $\displaystyle\int_2^8 \frac{1}{x} = \ln(x)\Big|_2^8 = \ln(8) - \ln(2) = \ln\left(\frac{8}{2}\right)$
$= \ln(4)$

b) $\displaystyle\int_6^1 \frac{1}{x} = \ln(x)\Big|_6^1 = \ln(1) - \ln(6)$
$= 0 - \ln(6)$ Rule ⑩
$= -\ln(6)$ Rule ⑧
$= \ln(6^{-1}) = \ln\left(\frac{1}{6}\right)$

c) $\dfrac{d}{dx}\left(\dfrac{-2x}{x+4}\right)^3 = 3\left(\dfrac{-2x}{x+4}\right)^2 \cdot \dfrac{d}{dx}\left(\dfrac{-2x}{x+4}\right)$ (Chain Rule)
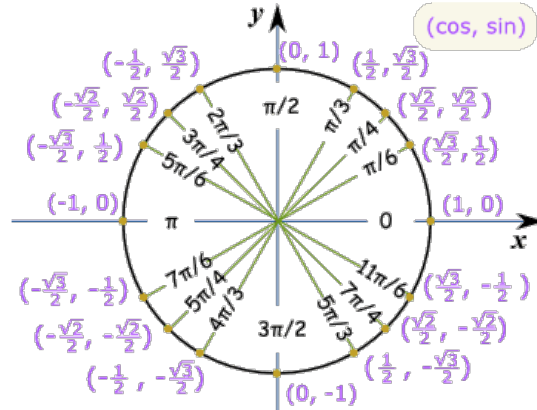
Raise num. & denom to 2 ↓   ↓ (Quotient Rule)

$3\dfrac{(-2x)^2}{(x+4)^2} \cdot \dfrac{(x+4)(-2) - (-2x)(1)}{(x+4)^2} =$

$\dfrac{3(-2x)^2}{(x+4)^2} \cdot \dfrac{(-8)}{(x+4)^2} = \dfrac{-24(-2x^2)}{(x+4)^6} = \boxed{\dfrac{48x^2}{(x+4)^6}}$ Rule ①

Things You Gotta Know: Common Trig Functions

Examples:

a) $\sin^2\left(\frac{\pi}{4}\right) = \left(\sin\frac{\pi}{4}\right)^2 = \left(\frac{\sqrt{2}}{2}\right)^2 = \frac{2}{4} = \frac{1}{2}$

b) $\cot\left(\frac{\pi}{2}\right) = \frac{\cos\left(\frac{\pi}{2}\right)}{\sin\left(\frac{\pi}{2}\right)} = \frac{0}{1} = 0$

c) $-\frac{\cos(0)}{5} = \frac{-1}{5}$

d) $\sec^2(\pi) = \frac{1}{\cos^2(\pi)} = \frac{1}{(-1)^2} = \frac{1}{1}$

e) $\displaystyle\int_0^{\pi/8} \sin(2x)\,dx = -\frac{1}{2}\cos(2x)\Big|_0^{\pi/8}$

$= -\frac{1}{2}\cos\left(2\cdot\frac{\pi}{8}\right) - \left(\frac{-1}{2}\cos(2\cdot 0)\right)$

$= -\frac{1}{2}\cos\left(\frac{\pi}{4}\right) + \frac{1}{2}\cos(0)$

$= -\frac{1}{2}\left(\frac{\sqrt{2}}{2}\right) + \frac{1}{2}(1)$

$= \frac{-\sqrt{2}}{4} + \frac{1}{2}$