

Integration techniques

Specific Functions

Radicals:

$$\int \sqrt{x} dx \quad \text{Switch to an exponent!}$$

$$\int x^{1/2} dx = \frac{x^{3/2}}{(3/2)} + C$$

\rightarrow multiply by reciprocal

$$= \frac{2}{3} x^{3/2} + C$$

Power Rule:

$$\int x^n dx = \frac{x^{n+1}}{n+1} + C$$

Examples:

$$\int x^4 dx = \frac{x^5}{5} + C$$

$$\int x^{2/3} dx = \frac{x^{5/3}}{5/3} + C$$

\rightarrow multiply by reciprocal

$$= \frac{3}{5} x^{5/3} + C$$

$$\int x^{-4} dx = \frac{x^{-3}}{-3} + C$$

Exponents & Logs:

$$\int e^x dx = e^x + C$$

$$\int \frac{1}{x} dx = \ln(x) + C$$

Trigonometric Rules:

$$\int \cos x dx = \sin x + C$$

$$\int \sin x dx = -\cos x + C$$

$$\int \sec^2 x dx = \tan x + C$$

$$\int -\csc^2 x dx = \cot x + C$$

$$\int \sec x \tan x dx = \sec x + C$$

$$\int -\csc x \cot x dx = \csc x + C$$

Inverse Trig:

$$\int \frac{1}{\sqrt{1-x^2}} dx = \sin^{-1}(x) + C$$

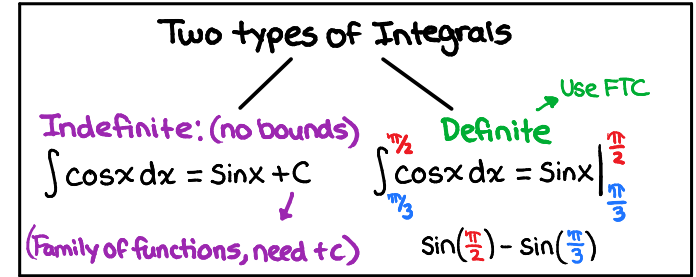
$$\int \frac{-1}{\sqrt{1-x^2}} dx = \cos^{-1}(x) + C$$

$$\int \frac{1}{1+x^2} dx = \tan^{-1}(x) + C$$

$$\int \frac{-1}{1+x^2} dx = \cot^{-1}(x) + C$$

$$\int \frac{1}{|x|\sqrt{x^2-1}} dx = \sec^{-1}(x) + C$$

$$\int \frac{-1}{|x|\sqrt{x^2-1}} dx = \csc^{-1}(x) + C$$



Fundamental Theorem of Calculus (FTC)

$$1) \frac{d}{dx} \int_a^x f(t) dt = f(x)$$

$$2) \int_a^b f(x) dx = F(b) - F(a)$$

where F is the anti derivative of $f(x)$

Integration with Substitution:

$$① \int_1^2 x^2(4x^3-2) dx \quad u=4x^3-2 \quad ① \text{ Make Substitution}$$

$$② \frac{du}{dx} = 12x^2 \quad ② \text{ Find } \frac{du}{dx}$$

$$③ \frac{du}{12x^2} = dx \quad ③ \text{ Solve for } dx$$

$$④ u=4(2)^3-2=30 \quad ④ \text{ Change bounds if definite integral}$$

$$u=4(1)^3-2=2$$

$$⑤ \int_2^{30} x^2(4x^3-2) \frac{du}{12x^2} \quad ⑤ \text{ Substitute and Simplify}$$

$$⑥ \frac{1}{12} \int_2^{30} u du = \frac{1}{12} \left[\frac{u^2}{2} \Big|_2^{30} \right]$$

$$= \frac{1}{12} \left[\frac{30^2}{2} - \frac{2^2}{2} \right] = \boxed{\frac{448}{12}}$$

⑥ Integrate and Evaluate