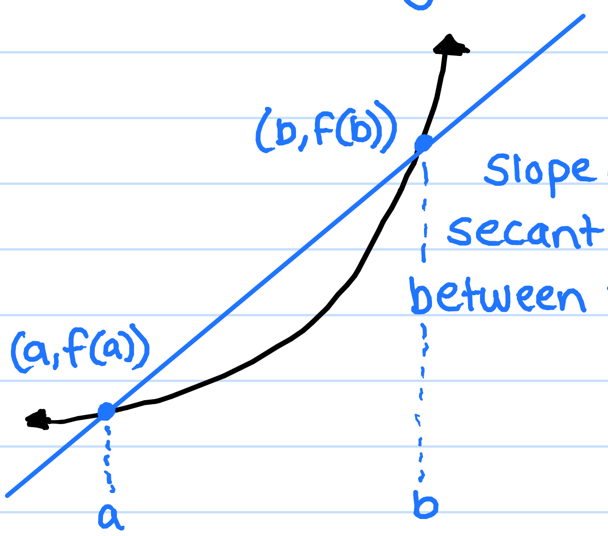
Mean Value Theorem

Recall:

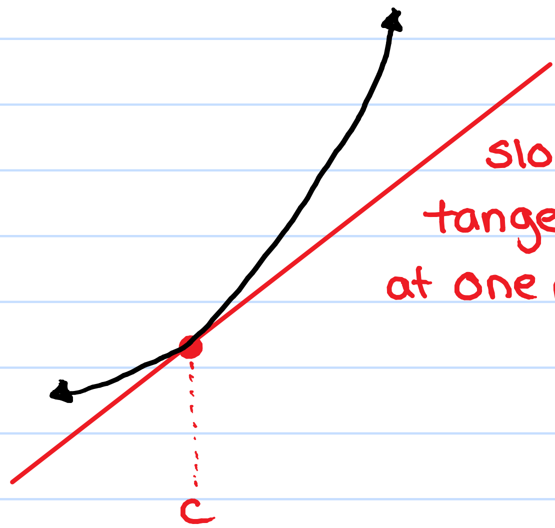
Average Rate of Change (AROC)



Slope of the secant line between 2 points

$$\text{slope} = \frac{f(b) - f(a)}{b - a}$$

Instantaneous Rate of Change (IRC)



slope of tangent line at one point.

slope $= f'(c)$

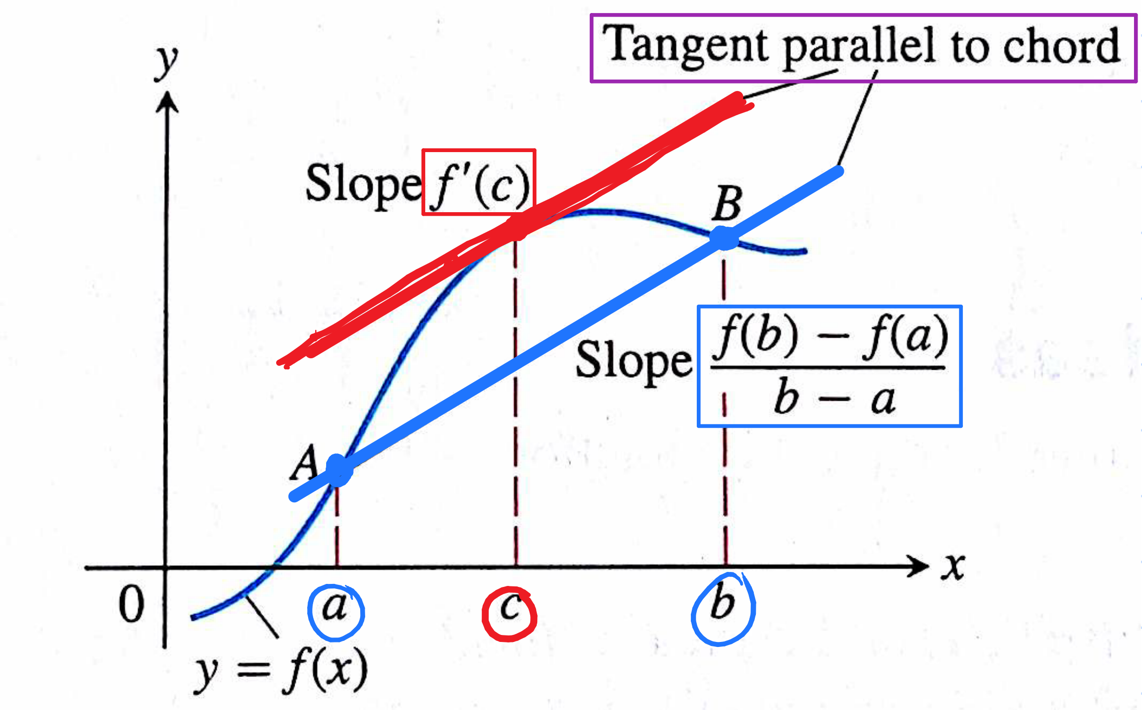
↳ CALCULUS

1st Derivative

Mean-Value Theorem: Let f(x) be a continuous on the interval [a,b] and differentiable on the interval (a,b). Then there is at least one point c in the interval (a,b) where the average rate of change equals the instantaneous rate of change:

$$f'(c) = \frac{f(b)-f(a)}{b-a}$$

Instantaneous Rate of Change — Average Rate of Change



Example 1: Assume $f(x)$ is differentiable on the interval $[8, 24]$

x	8	12	20	24
$f(x)$	-12	48	60	16

a) Show that there exists a value c_1 on the interval $[8,12]$ such that $f(c_1)=0$

b) Show that there exists a value c_2 on the interval $[8,12]$ such that $f'(c_2)=15$.

c) Show that there exists a value c_3 on the interval $[20,24]$ such that $f'(c_3)=-11$

Example 2: A car driver takes 2 hours to travel from two toll boths that are 159 miles apart. If the speed limit on the highway is 65 MPH, should the driver receive a speeding ticket? Why or why not?

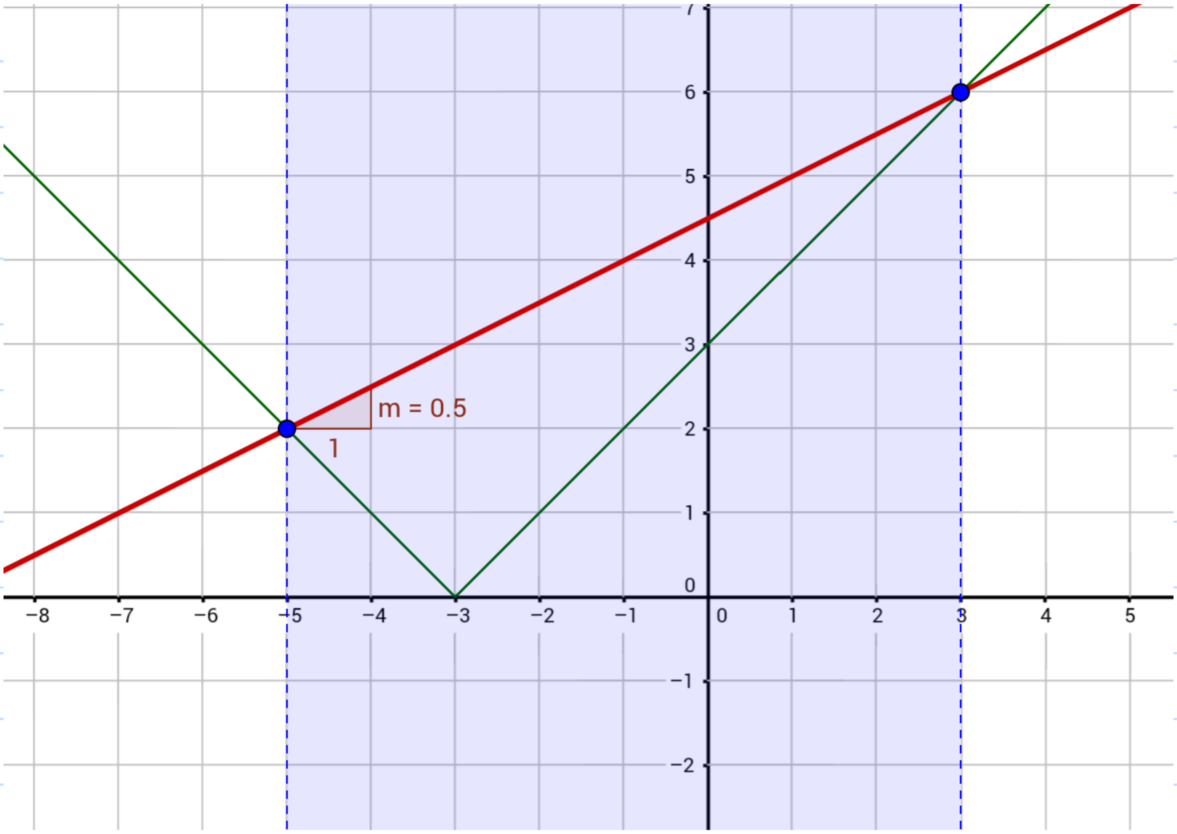
Example 3: Mr. Cat says the function $f(x) = |x+3|$ must have at least one point c for which $f'(c) = \frac{1}{2}$ on the interval $(-5,3)$. His work is show below:

$f(-5) = |-5+3| = |-2| = 2$
$f(3) = |3+3| = |6| = 6$

Avg Rate of Change $= \frac{6-2}{3-(-5)} = \frac{4}{8} = \frac{1}{2}$

$\therefore$ By the Mean Value Theorem, there exists a point c on $(-5,3)$ where $f'(c) = 1/2$

What is wrong with Mr. Cat's reasoning?



m = 0.5
1