

Master Problem Set R2

1. Solve the following systems algebraically

a)
 $y = 5x^2 - 5x + 3$
 $y = 2x + 9$

$$\begin{aligned} 2x + 9 &= 5x^2 - 5x + 3 \\ -2x &\quad -2x \\ 9 &= 5x^2 - 7x + 3 \\ -9 &\quad -9 \\ 0 &= 5x^2 - 7x - 6 \end{aligned}$$

$$\begin{array}{r} -7 \\ \times \\ -7 \\ +3 \\ \hline -36 \end{array}$$

$$\begin{aligned} 0 &= (5x^2 - 10x) + (3x - 6) \\ 0 &= 5x(x-2) + 3(x-2) \\ 0 &= (5x+3)(x-2) \end{aligned}$$

$$\left(-\frac{3}{5}, \frac{39}{5}\right)$$

$$(2, 13)$$

$$\begin{array}{r} 5x+3=0 \quad x-2=0 \\ -3 \quad -3 \quad +2 \quad +2 \\ \hline \frac{5x}{5} = \frac{-3}{5} \quad x=2 \end{array}$$

Both solved for y
 $5x^2 - 10x + 3 = 2x + 9$
 $5x^2 - 12x - 6 = 0$
 $(5x+3)(x-2) = 0$
 $5x+3=0 \Rightarrow x = -\frac{3}{5}$
 $x-2=0 \Rightarrow x = 2$
 The solutions are $(-\frac{3}{5}, \frac{39}{5})$ and $(2, 13)$

b)
 $y = -x^2 - 3x + 41$
 $y = -3x - 8$

$$\begin{aligned} -3x - 8 &= -x^2 - 3x + 41 \\ +3x &\quad +3x \end{aligned}$$

$$\begin{aligned} -8 &= -x^2 + 41 \\ -41 &\quad -41 \end{aligned}$$

$$\begin{aligned} -49 &= -x^2 \\ -1 &\quad -1 \end{aligned}$$

$$\begin{aligned} 49 &= x^2 \\ x &= \pm 7 \end{aligned}$$

$$(-7, 13)$$

$$(7, -29)$$

c)
 $(x+2)^2 + y^2 = 40$
 $x - y = 2$

$$\begin{aligned} (x+2)^2 + y^2 &= 40 \\ (y+2+2)^2 + y^2 &= 40 \\ (y+4)^2 + y^2 &= 40 \\ y^2 + 8y + 16 + y^2 &= 40 \\ \frac{2y^2}{2} + \frac{8y}{2} - \frac{24}{2} &= 0 \end{aligned}$$

$$\begin{aligned} y^2 + 4y - 12 &= 0 \\ (y+6)(y-2) &= 0 \\ y &= -6 \\ y &= 2 \end{aligned}$$

$$(-4, -6)$$

$$(4, 2)$$

Solve for x
 $x - y = 2 \Rightarrow x = y + 2$

d)
 $x^2 + (y-1)^2 = 97$
 $-x + y = 6$

$$\begin{aligned} x^2 + (y-1)^2 &= 97 \\ x^2 + (x+6-1)^2 &= 97 \\ x^2 + (x+5)^2 &= 97 \\ x^2 + x^2 + 10x + 25 &= 97 \\ \frac{2x^2}{2} + \frac{10x}{2} - \frac{72}{2} &= 0 \end{aligned}$$

$$\begin{aligned} x^2 + 5x - 36 &= 0 \\ (x+9)(x-4) &= 0 \\ x &= -9 \quad x = 4 \end{aligned}$$

$$(-9, -3)$$

$$(4, 10)$$

e)
 $(x+5)^2 + y^2 = 13$
 $x - 3y = -2$

$$\begin{aligned} (3y-2+5)^2 + y^2 &= 13 \\ (3y+3)^2 + y^2 &= 13 \\ 9y^2 + 18y + 9 + y^2 &= 13 \\ \frac{10y^2}{2} + \frac{18y}{2} - \frac{4}{2} &= 0 \end{aligned}$$

$$\left(-\frac{7}{5}, \frac{1}{5}\right)$$

$$(-8, -2)$$

We know y, so find x
 $x - 3y = -2$
 $x - 3(-\frac{7}{5}) = -2$
 $x + \frac{21}{5} = -2$
 $x = -\frac{21}{5} - 2 = -\frac{21}{5} - \frac{10}{5} = -\frac{31}{5}$
 $(-\frac{31}{5}, -\frac{7}{5})$
 $x - 3y = -2$
 $x - 3(-8) = -2$
 $x + 24 = -2$
 $x = -24 - 2 = -26$
 $(-26, -8)$

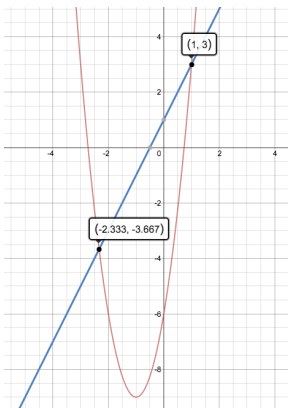
$$\begin{aligned} 5y^2 + 9y - 2 &= 0 \\ (5y^2 + 10y) - 1(y-2) &= 0 \\ 5y(y+2) - 1(y-2) &= 0 \\ (5y-1)(y+2) &= 0 \\ y &= \frac{1}{5} \quad y = -2 \end{aligned}$$

2. Solve the following systems graphically (round all decimals to nearest tenth):

a)

$$y = 3x^2 + 6x - 6$$

$$y = 2x + 1$$



Solutions: (1, 3)
and (-2.333, -3.667)

b)

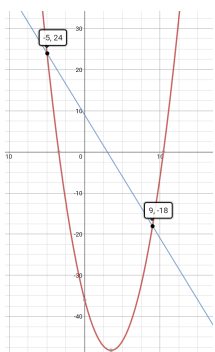
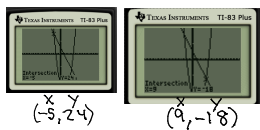
$$y = x^2 - 7x - 36$$

$$3x + y = 9$$

$$-3x \quad -3x$$

$$y = -3x + 9$$

Graphically: Always solve for y

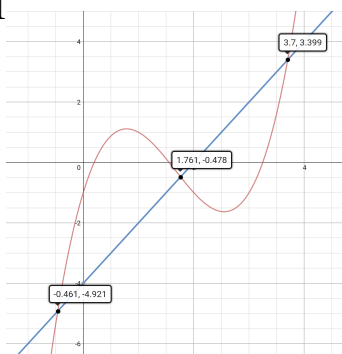


Solutions: (-5, 24)
and (9, -18)

c)

$$y = x^3 - 5x^2 + 6x - 1$$

$$y = 2x - 4$$

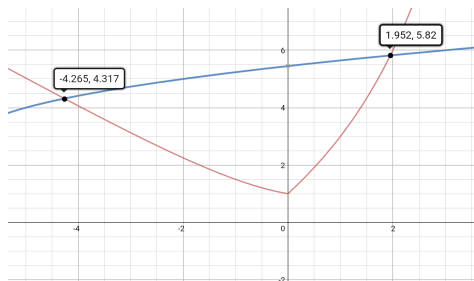


Answers:
(3.7, 3.4)
(1.8, -0.5)
(-0.5, -4.9)

d)

$$y = |x| + 2^x$$

$$y = \sqrt{x+6} + 3$$

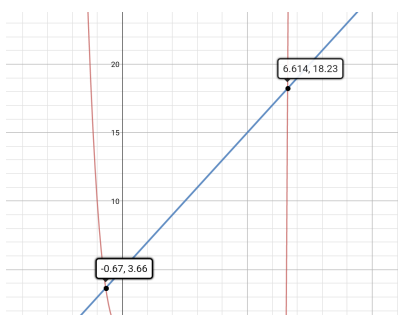


Solutions:
(2.0, 5.8) and
(-4.3, 4.3)

e)

$$y = x^4 - \frac{13}{2}x^3 - \frac{9}{4}x$$

$$y = 2x + 5$$



Solutions:
(6.6, 18.2)
and
(-.7, 3.7)

3. Solve the following system algebraically:



a)

$$\begin{aligned} 9x + 9y - 8z &= 10 \\ 3x + 4y - 3z &= 9 \\ 9x + 6y + 2z &= 2 \end{aligned}$$

Handwritten solution for part a):

Original system:

$$\begin{aligned} 9x + 9y - 8z &= 10 \\ 3x + 4y - 3z &= 9 \\ 9x + 6y + 2z &= 2 \end{aligned}$$

Step 1: Eliminate x from equations 2 and 3.

$$\begin{aligned} (3x + 4y - 3z = 9) \times 3 &\rightarrow 9x + 12y - 9z = 27 \\ (9x + 6y + 2z = 2) &\rightarrow 9x + 6y + 2z = 2 \end{aligned}$$

$$\begin{aligned} (9x + 12y - 9z = 27) - (9x + 6y + 2z = 2) &\rightarrow 6y - 11z = 25 \end{aligned}$$

Step 2: Eliminate x from equation 1.

$$(9x + 9y - 8z = 10) - (9x + 6y + 2z = 2) \rightarrow 3y - 10z = 8$$

Step 3: Eliminate y from the two new equations.

$$\begin{aligned} (3y - 10z = 8) \times 2 &\rightarrow 6y - 20z = 16 \\ (6y - 11z = 25) &\rightarrow 6y - 11z = 25 \end{aligned}$$

$$\begin{aligned} (6y - 20z = 16) - (6y - 11z = 25) &\rightarrow -9z = -9 \\ z &= 1 \end{aligned}$$

Step 4: Substitute z = 1 back into the equations.

$$\begin{aligned} 3y - 10(1) &= 8 \rightarrow 3y - 10 = 8 \rightarrow 3y = 18 \rightarrow y = 6 \\ 9x + 9(6) - 8(1) &= 10 \rightarrow 9x + 54 - 8 = 10 \rightarrow 9x + 46 = 10 \rightarrow 9x = -36 \rightarrow x = -4 \end{aligned}$$

Final solution: $(-4, 6, 1)$

$(-4, 6, 1)$

b)

$$\begin{aligned} -2x - 5y + 3z &= 10 \\ x + 9y + 5z &= -5 \\ 4x - y - 8z &= 7 \end{aligned}$$

$(7, -3, 3)$

c)

$$\begin{aligned} 8x + 2y + 9z &= -6 \\ 4x + 3y + 2z &= -4 \\ -4x + 3y - 8z &= -8 \end{aligned}$$

$(8, -8, -6)$

d)

$$\begin{aligned} 7x + 4y + 9z &= -5 \\ 4x + 5y - 9z &= -4 \\ 4x + 6y - 9z &= 6 \end{aligned}$$

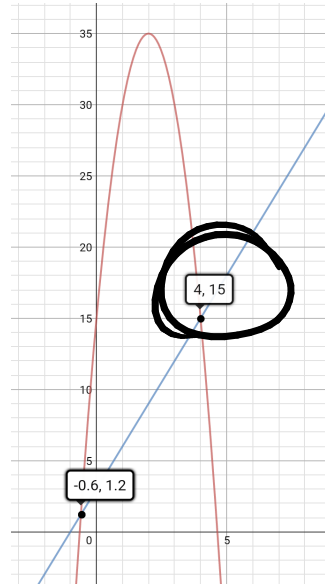
$(-9, 10, 2)$

e)

A candy store sells three different packages of candy: Packages of lollipops, gum, and chocolate. Sarai comes in and purchases 9 packages of lollipops, 3 packages of gum, and 4 packages of chocolate and her total is \$53. Benji purchases 9 packages of lollipops, 6 packages of gum, and 5 packages of chocolate and spends \$64. Giselle buys three packages of lollipops, 5 packages of gum, and 2 packages of chocolate and her total is \$31. Find the total cost of each package of candy.

Lollipops: \$4 per package
 Gum: \$3 per package
 Chocolate: \$2 per package

f) Yes, they will meet
after 4 seconds at a
spot 15 feet above the
ground



g) System: $v = \#$ of volleyballs sold
 $b = \#$ of basketballs sold
 $f = \#$ of footballs sold

$$5v = f$$

$$35f + 25b + 15v = 3750$$

$$\begin{aligned} 4(15v) &= 25b \\ \rightarrow 60v &= 25b \end{aligned}$$

① Substitute $5v$ in for f in second equation

$$35(5v) + 25b + 15v = 3750$$

$$175v + 25b + 15v = 3750$$

② substitute $60v$ in for $25b$

$$175v + (60v) + 15v = 3750$$

$$\frac{250v}{250} = \frac{3750}{250}$$

$$v = 15$$

$$5(15) = f$$

$$75 = f$$

$$60(15) = 25b$$

$$900 = 25b$$

$$\frac{900}{25} = \frac{25b}{25}$$

$$36 = b$$