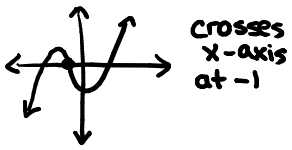
Master Problem Set R1:

1. Consider the polynomial $P(x) = x^3 + x^2 - 4x - 4$

a) Show that -1 is a zero of this polynomial in at least two different ways:

$P(-1) = (-1)^3 + (-1)^2 - 4(-1) - 4$
$= 0$



b) Since -1 is a zero, what is one of the **factors** of this polynomial?

-1 is a zero means that (x+1) is a factor

$(x^3 + x^2 - 4x - 4)$
$x^2(x+1) - 4(x+1)$
$(x^2 - 4)(x+1)$
$(x-2)(x+2)(x+1)$

c) Find all of the factors of the polynomial (think about how you could factor this). Your final answer should be in the form:

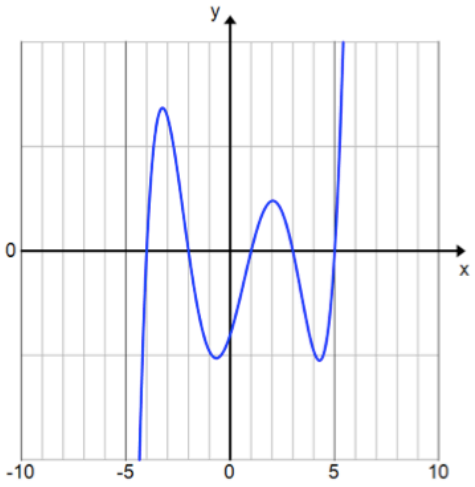
$(x - 2)(x + 2)(x + 1)$

d) Based on the factors, find all of the zeros of the polynomial:

Factors: $(x-2)(x+2)(x+1)$
↓ ↓ ↓
Zeros: 2 −2 −1

2. Consider the graph of a degree 5 polynomial shown below, with x-intercepts -4, -2, 1, 3 and 5.

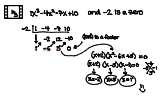
a) Write a factored polynomial $f(x)$ that could represent the function based on its zeros.



-4 -2 1 3 5 Zeroes

$f(x) = (x+4)(x+2)(x-1)(x-3)(x-5)$ factors

3. If $p(x) = x^3 - 4x^2 - 7x + 10$ has a zero of -2. Find all other zeros of the polynomial using synthetic division:



$x^3 \quad -4x^2 \quad -7x \quad +10$

$$\begin{array}{r|cccc} -2 & 1 & -4 & -7 & 10 \\ & \downarrow & -2 & 12 & -10 \\ \hline & 1 & -6 & 5 & 0 \end{array}$$

$(x+2)(x^2-6x+5)=0$

$(x+2)(x-5)(x-1)=0$

$x=-2 \quad x=5 \quad x=1$

$-5 \times -1 = 5$, $-5 + -1 = -6$

Zeros:
$x=-2$
$x=5$
$x=1$

4. If $f(x) = x^3 + 12x^2 + 29x + 18$ and -1 is a zero, find all the solutions to the equation $x^3 + 12x^2 + 29x + 18 = 0$

$x^3 \quad +12x^2 \quad +29x \quad +18$

$$\begin{array}{r|cccc} -1 & 1 & 12 & 29 & 18 \\ & \downarrow & -1 & -11 & -18 \\ \hline & 1 & 11 & 18 & 0 \end{array}$$

$(x+1)(x^2+11x+18)=0$

$(x+1)(x+2)(x+9)=0$

$x=-1 \quad x=-2 \quad x=-9$

$2 \times 9 = 18$, $2 + 9 = 11$

Zeros:
$x=-1$
$x=-2$
$x=-9$

5. If $f(x) = x^3 - 5x^2 - 41x + 45$ and $f(-5) = 0$, then find all of the zeros algebraically: **Draw a rough sketch of the polynomial, showing the zeros!**

$x^3 \quad -5x^2 \quad -41x \quad +45$

$$\begin{array}{r|cccc} -5 & 1 & -5 & -41 & 45 \\ & \downarrow & -5 & 50 & -45 \\ \hline & 1 & -10 & 9 & 0 \end{array}$$

$(x+5)(x^2-10x+9)=0$

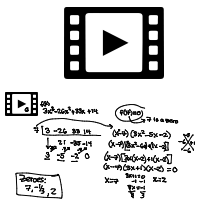
$(x+5)(x-9)(x-1)=0$

$x=-5 \quad x=9 \quad x=1$

$-9 \times -1 = 9$, $-9 + -1 = -10$

Zeros:
$x=-5$
$x=9$
$x=1$

6. If $f(x) = 3x^3 - 26x^2 + 33x + 14$ and $f(7) = 0$, then find all the zeros of $f(x)$ algebraically. **Draw a rough sketch of the polynomial, showing the zeros!**



$3x^3 \quad -26x^2 \quad +33x \quad +14$

$$\begin{array}{r|cccc} 7 & 3 & -26 & 33 & 14 \\ & \downarrow & 21 & -35 & -14 \\ \hline & 3 & -5 & -2 & 0 \end{array}$$

$(x-7)(3x^2-5x-2)=0$

$(x-7)[(3x^2-6x)+(1x-2)]=0$

$(x-7)[3x(x-2)+1(x-2)]=0$

$(x-7)(3x+1)(x-2)=0$

$x=7 \quad x=-\frac{1}{3} \quad x=2$

$-6 \times +1 = -6$, $-6 + +1 = -5$

Zeros:
$x=7$
$x=-1/3$
$x=2$

7. If $g(x) = 2x^3 + 3x^2 - 3x - 2$ and $x + 2$ is a factor of $f(x)$, then find all of the zeros of $g(x)$

$2x^3 + 3x^2 \quad -3x \quad -2$

$$\begin{array}{r|cccc} -2 & 2 & 3 & -3 & -2 \\ & \downarrow & -4 & 2 & 2 \\ \hline & 2 & -1 & -1 & 0 \end{array}$$

$(x+2)(2x^2-x-1)=0$

$(x+2)[(2x^2-2x)+(x-1)]=0$

$(x+2)[2x(x-1)+1(x-1)]=0$

$(x+2)(2x+1)(x-1)=0$

$x=-2 \quad x=-\frac{1}{2} \quad x=1$

$-2 \times +1 = -2$, $-2 + +1 = -1$

Zeros:
$x=-2$
$x=-1/2$
$x=1$

8. If $f(x) = 3x^3 - x^2 - 27x + 9$ and $x + 3$ is a factor of $f(x)$, then find all zeros of $f(x)$

$3x^3 - x^2 - 27x + 9$

$-3 \mid 3 \quad -1 \quad -27 \quad 9$

$\quad\quad -9 \quad 30 \quad -9$

$\quad 3 \quad -10 \quad 3 \quad 0$

$(x+3)(3x^2-10x+3)=0$

$(x+3)(3x^2-9x-x+3)=0$

$(x+3)[3x(x-3)-1(x-3)]=0$

$(x+3)(3x-1)(x-3)=0$

$x=-3 \quad x=\frac{1}{3} \quad x=3$

$-10 = -9 \cdot -1$

Zeroes:

$x=3$

$x=\frac{1}{3}$

$x=3$

9. Find all zeros of the polynomial (hint: is it possible to factor this?)

$$4x^3 - 28x^2 - 25x + 175 = 0$$

Factor by grouping!

$(4x^3-28x^2)+(-25x+175)=0$

$4x^2(x-7)-25(x-7)=0$

$(4x^2-25)(x-7)=0$

DOPS

$(4x^2-25)(x-7)=0$

$(2x-5)(2x+5)(x-7)=0$

$x=\frac{5}{2} \quad x=-\frac{5}{2} \quad x=7$

10. If $f(x) = x^3 - 3x + k$ and $x + 3$ is a factor of $f(x)$, find the value of k:

$x^3 + 0x^2 - 3x + k$

$-3 \mid 1 \quad 0 \quad -3 \quad k$

$\quad\quad -3 \quad 9 \quad -18$

$\quad 1 \quad -3 \quad 6 \quad (k-18)$

This must be zero in order for x+3 to be a factor:

$k-18=0$

$k=18$

11. Given $f(x) = 2x^3 + kx + 6$, and $x + 2$ is a factor of f(x), then what is the value of *k*?

$2x^3+0x^2+kx+6$

$-2 \mid 2 \quad 0 \quad k \quad 6$

$\quad\quad -4 \quad 8 \quad -2k-16$

$\quad 2 \quad -4 \quad k+8 \quad -2k-10$

This must be zero in order for x+3 to be a factor:

$-2k-10=0$

$\frac{-2k}{-2} = \frac{10}{-2}$

$k=-5$

12. Fully factor the following polynomials:

a) $x^4 - 13x^2 + 40$ Let $y = x^2$

$y^2 - 13y + 40$

-13: $-5, -8$ ($-5 \cdot -8 = 40$)

$(y-5)(y-8)$

$(x^2-5)(x^2-8)$

b) $9r^4 - 13r^3 + 4r^2$ GCF first

$r^2(9r^2 - 13r + 4)$

$r^2(9r^2 - 4r - 9r + 4)$

$r^2(r(9r-4) - 1(9r-4))$

$r^2(r-1)(9r-4)$

-13: $-4, -9$ ($-4 \cdot -9 = 36$)

c) $-4x^6 + 3x^5 + 16x^4 - 12x^3$ GCF first!

$-x^3(4x^3 - 3x^2 - 16x + 12)$ Grouping!

$-x^3[x^2(4x-3) - 4(4x-3)]$

$(-x^3)(x^2-4)(4x-3)$ → DOPS!

$(-x^3)(x-2)(x+2)(4x-3)$

d) $(5x^2 + 11x)^2 - 6(5x^2 + 11x) - 72$ Let $y = 5x^2 + 11x$

$y^2 - 6y - 72$

-6: $-12, 6$ ($-12 \cdot 6 = -72$)

$(y-12)(y+6)$

$(5x^2+11x-12)(5x^2+11x+6)$ Factor each by super magic X

$(5x-4)(x+3)(5x+6)(x+1)$

e) $(x^2 - 5x)^2 - 8(x^2 - 5x) - 84$ Let $y = x^2 - 5x$

$y^2 - 8y - 84$

-8: $-14, +6$ ($-14 \cdot 6 = -84$)

$(y-14)(y+8)$

$(x^2-5x-14)(x^2-5x+8)$

-5: $-7, +2$ ($-7 \cdot 2 = -14$)

$(x-7)(x+2)(x^2-5x+8)$

f) $x^4 - y^4$

$a = x^2$
$b = y^2$

$a^2 - b^2$

$(a-b)(a+b)$

$(x^2-y^2)(x^2+y^2)$

$(x-y)(x+y)(x^2+y^2)$

g) $(2x+3)^2 - (3x+1)^2$

$a = 2x+3$
$b = 3x+1$

$a^2 - b^2$

$(a-b)(a+b)$

$(2x+3-(3x+1))\ (2x+3+(3x+1))$

Combine like terms in each factor

$(-x+2)(5x+4)$

13. Factor the following fully:

a) $(2x-3)(4x+5) - (4x+5)(5x-1)$

Let $a = 4x+5$

$(2x-3)a - a(5x-1)$

$a[(2x-3)-(5x-1)]$

$a(-3x-2)$

$(4x+5)(-3x-2)$

b) $[x^4 - x^2][+5x^3 - 5x][+4x^2 - 4]$ Factor by grouping!

$x^2(x^2-1) + 5x(x^2-1) + 4(x^2-1)$

magic X → $(x^2+5x+4)(x^2-1)$ → DOPS

$(x+4)(x+1)(x-1)(x+1)$

c) $2y^4 - 19y^2g^2 + 44g^4$

-19, -8, -11, 88

$(2y^4-8y^2g^2)(-11y^2g^2+44g^4)$

$2y^2(y^2-4g^2) - 11g^2(y^2-4g^2)$

$(2y^2-11g^2)(y^2-4g^2)$

$(2y^2-11g^2)(y-2g)(y+2g)$

d) $10c^4 + 11c^2t^4 - 39t^8$

$10c^4 + 26c^2t^4 - 15c^2t^4 - 39t^8$

11, 26, -15, -390

$2c^2(5c^2+13t^4) - 3t^4(5c^2+13t^4)$

$(2c^2-3t^4)(5c^2+13t^4)$

13. Fully factor the polynomials below:

a) $[x^3 + x^2] + [2x^2 + 2x] - [3x - 3]$

$x^2(x+1) + 2x(x+1) - 3(x+1)$

$(x^2+2x-3)(x+1)$

$3 \times -1 = -3$, 2

$(x+3)(x-1)(x+1)$

b) $(2x+3)^2 + 6(2x+3) + 8$

$y = 2x+3$

$y^2 + 6y + 8$

$6 = 2 \times 4$, $2 + 4 = 6$

$(y+2)(y+4)$

$(2x+3+2)(2x+3+4)$

$(2x+5)(2x+7)$

c) $(x-4)^2 - 5(x-4) - 14$

$y = x-4$

$y^2 - 5y - 14$

$-5 = -7 + 2$, $-7 \times 2 = -14$

$(y-7)(y+2)$

$(x-4-7)(x-4+2)$

$(x-11)(x-2)$

d) $(a+2)^2 - (b+2)^2$

$x = a+2$

$y = b+2$

$x^2 - y^2$

$(x-y)(x+y)$

$(a+2-(b+2))(a+2+(b+2))$

$(a-b)(a+b+4)$

e) $25 - (x^2 - 6x + 9)$

$25 - x^2 + 6x - 9$

$-x^2 + 6x + 16$

$-6 = -8 + 2$, $-8 \times 2 = -16$

$-(x^2 - 6x - 16)$

$-(x-8)(x+2)$

10, 5, 5, 25   -12, -6, -6, 36

f) $(a^2 + 10a + 25) - (x^2 - 12x + 36)$

$(a+5)^2 - (x-6)^2$ $y = a+5$, $z = x-6$

$y^2 - z^2$

$(y-z)(y+z)$

$[a+5-(x-6)][a+5+(x-6)]$

$(a-x+11)(a+x-1)$

g) Find all real solutions to $x^3 - 5x^2 - 4x + 20 = 0$.

$(x^3-5x^2)(-4x+20) = 0$

$x^2(x-5) - 4(x-5) = 0$

$(x^2-4)(x-5) = 0$

$(x-2)(x+2)(x-5) = 0$

$x=2 \quad x=-2 \quad x=5$

h) Find all real solutions to $x^3 - 3x^2 - 4x + 12 = 0$.

$(x^3-3x^2)(-4x+12) = 0$

$x^2(x-3) - 4(x-3) = 0$

$(x^2-4)(x-3) = 0$

$(x-2)(x+2)(x-3) = 0$

$x=2 \quad x=-2 \quad x=3$

Master Problem Set R2

1. Solve the following systems algebraically

a)

$y = 5x^2 - 5x + 3$

$y = 2x + 9$

$2x+9 = 5x^2-5x+3$

$-2x \quad\quad -2x$

$9 = 5x^2-7x+3$

$-9 \quad\quad -9$

$0 = 5x^2-7x-6$

$-10, +3, -7, -36$

$0 = (5x^2-10x)+(3x-6)$

$0 = 5x(x-2)+3(x-2)$

$0 = (5x+3)(x-2)$

$5x+3=0 \quad x-2=0$

$-3\ -3 \quad +2\ +2$

$\frac{5x}{5} = \frac{-3}{5} \quad x=2$

$\left(-\frac{3}{5}, \frac{39}{5}\right)$

$(2, 13)$

b)

$y = -x^2 - 3x + 41$

$y = -3x - 8$

$-3x-8 = -x^2-3x+41$

$+3x \quad\quad +3x$

$-8 = -x^2+41$

$-41 \quad\quad -41$

$\frac{-49}{-1} = \frac{-x^2}{-1}$

$49 = x^2$

$x = \pm 7$

$(-7, 13)$

$(7, -29)$

c)

$(x+2)^2 + y^2 = 40$

$x - y = 2$

Solve for x: $+y \quad +y$

$x = y+2$

$(x+2)^2 + y^2 = 40$

$(y+2+2)^2 + y^2 = 40$

$(y+4)^2 + y^2 = 40$

$y^2+8y+16+y^2 = 40$

$\frac{2y^2}{2} + \frac{8y}{2} - \frac{24}{2} = 0$

$y^2+4y-12=0$

$4, -2, 6, -12$

$(y+6)(y-2)=0$

$y=-6$

$y=2$

$(-4, -6)$

$(4, 2)$

d)

$x^2 + (y-1)^2 = 97$

$-x + y = 6$

$+x \quad\quad +y$

$y = x+6$

$x^2+(y-1)^2 = 97$

$x^2+(x+6-1)^2 = 97$

$x^2+(x+5)^2 = 97$

$x^2+x^2+10x+25 = 97$

$\frac{2x^2}{2} + \frac{10x}{2} - \frac{72}{2} = 0$

$x^2+5x-36=0$

$5, -4, 9, -36$

$(x+9)(x-4)=0$

$x=-9 \quad x=4$

$(-9, -3)$

$(4, 10)$

e)

$(x+5)^2 + y^2 = 13$

$x - 3y = -2$

$x = 3y-2$

$(3y-2+5)^2+y^2=13$

$(3y+3)^2+y^2=13$

$9y^2+18y+9+y^2=13$

$\frac{10y^2}{2} + \frac{18y}{2} - \frac{4}{2} = 0$

$5y^2+9y-2=0$

$(5y^2+10y)(-1y-2)=0$

$5y(y+2)-1(y+2)=0$

$(5y-1)(y+2)=0$

$9, -1, 10, -10$

$y=\frac{1}{5} \quad y=-2$

$\left(-\frac{7}{5}, \frac{1}{5}\right)$

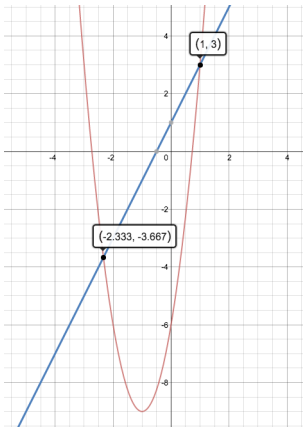
$(-8, -2)$

2. Solve the following systems graphically (round all decimals to nearest tenth):

a)

$y = 3x^2 + 6x - 6$

$y = 2x + 1$



Solutions: (1,3) and (-2.333, -3.667)

b)

2b

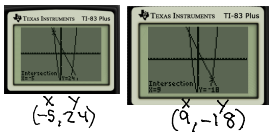
$y = x^2 - 7x - 36$

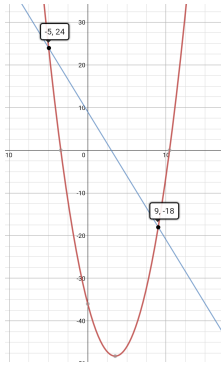
$3x + y = 9$

$-3x \quad -3x$

$y = -3x + 9$

Graphically: Always solve for y



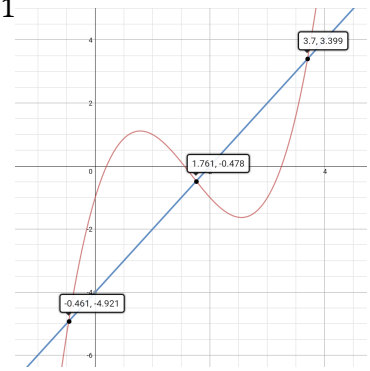


Solutions: (-5, 24) and (9, -18)

c)

$y = x^3 - 5x^2 + 6x - 1$

$y = 2x - 4$

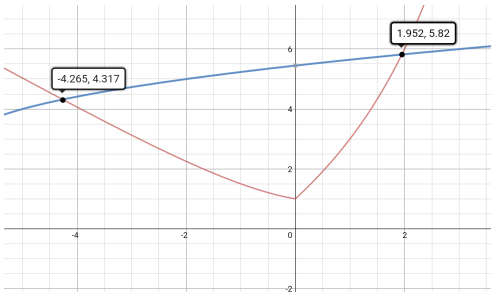


Answers: (3.7, 3.4) (1.8, -0.5) (-0.5, -4.9)

d)

$y = |x| + 2^x$

$y = \sqrt{x + 6} + 3$

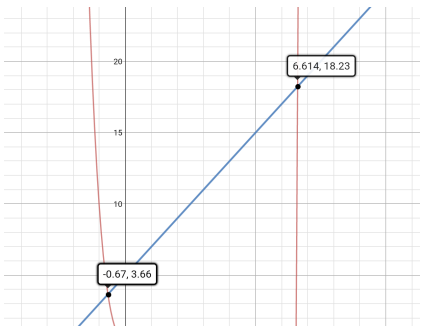


Solutions: (2.0, 5.8) and (-4.3, 4.3)

e)

$y = x^4 - \frac{13}{2}x^3 - \frac{9}{4}x$

$y = 2x + 5$

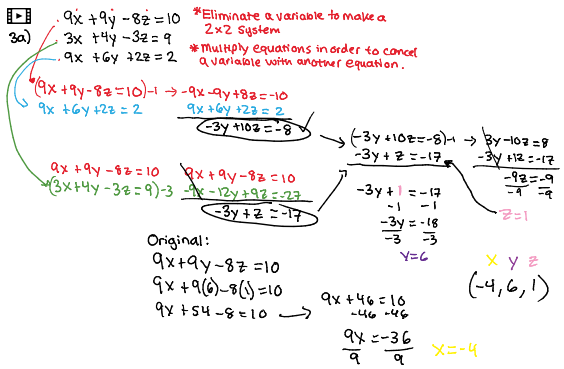


Solutions: (6.6, 18.2) and (-.7, 3.7)

3. Solve the following system algebraically:

a)

$9x + 9y - 8z = 10$
$3x + 4y - 3z = 9$
$9x + 6y + 2z = 2$



(-4, 6, 1)

b)

$-2x - 5y + 3z = 10$
$x + 9y + 5z = -5$
$4x - y - 8z = 7$

(7, -3, 3)

c)

$8x + 2y + 9z = -6$
$4x + 3y + 2z = -4$
$-4x + 3y - 8z = -8$

(8, -8, -6)

d)

$7x + 4y + 9z = -5$
$4x + 5y - 9z = -4$
$4x + 6y - 9z = 6$

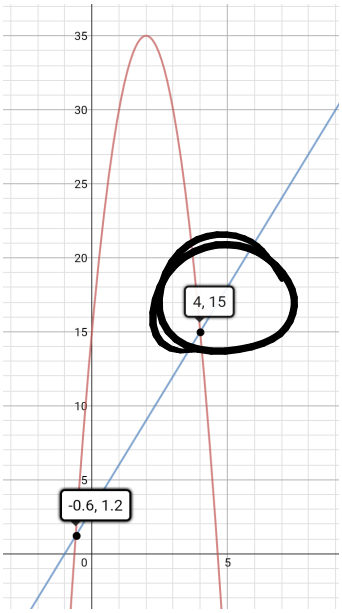
(-9, 10, 2)

e)

A candy store sells three different packages of candy: Packages of lollipops, gum, and chocolate. Sarai comes in and purchases 9 packages of lollipops, 3 packages of gum, and 4 packages of chocolate and her total is $53. Benji purchases 9 packages of lollipops, 6 packages of gum, and 5 packages of chocolate and spends $64. Giselle buys three packages of lollipops, 5 packages of gum, and 2 packages of chocolate and her total is $31. Find the total cost of each package of candy.

Lollipops: $4 per package
Gum: $3 per package
Chocolate: $2 per package

f) Yes, they will meet after 4 seconds at a spot 15 feet above the ground



g) System: v = # of volleyballs sold
b = # of basketballs sold
f = # of footballs sold

$5v = f$

$35f + 25b + 15v = 3750$

$4(15v) = 25b$

$60v = 25b$

① Substitute $5v$ in for f in second equation

$35(5v) + 25b + 15v = 3750$

$175v + 25b + 15v = 3750$

② Substitute $60v$ in for $25b$

$175v + (60v) + 15v = 3750$

$\frac{250v}{250} = \frac{3750}{250}$

$v = 15$

$5(15) = f$

$75 = f$

$60(15) = 25b$

$\frac{900}{25} = \frac{25b}{25}$

$36 = b$