Name: **Date:** **Period:**

Inverses and Transformations of Exponential and Logarithmic Functions

____LG 6.2: Investigate and apply previously learned rules for transforming functions and exponential and logarithmic functions and make connections to different functions.

Part 1: Inverses:

We can calculate the inverse of an exponential and logarithmic function algebraically:

Example1: Find the inverse of the function $y = \log_2 x$:

$\boldsymbol{y = \log_2 x}$

$\boldsymbol{x = \log_2 y}$

$\boldsymbol{y = 2^x}$

Example 2: Find the inverse of the function $y = 3.4^x$

$\boldsymbol{y = 3.4^x}$

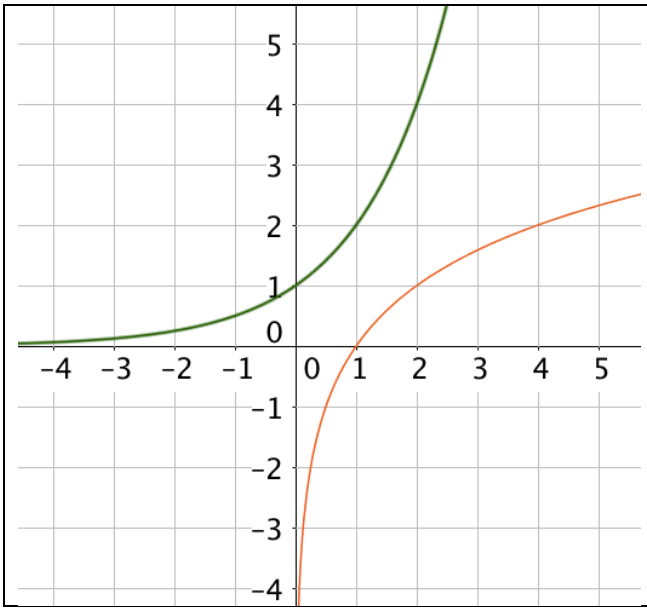
$\boldsymbol{x = 3.4^y}$

$\boldsymbol{y = \log_{3.4} x}$

1: Find the inverse of the following functions:

$\boldsymbol{y = 4^x}$ $\boldsymbol{y = \log_6 x}$ $\boldsymbol{y = 1.08^x}$

Part 2: Parent Exponential and Logarithmic Functions:



PARENT Exponential Graphs: $y = a^x$

- **y-intercept at (0,1)**
- **No x-intercept**
- **Asymptote at $y = 0$ (x-axis)**
  - **This means that the graph will never touch the x-axis.**

PARENT Logarithmic Graphs: $y = \log_a x$

- **x-intercept at (1,0)**
- **No y-intercept**
- **Asymptote at $x = 0$ (y-axis)**
  - **This means that the graph will never touch the y-axis.**

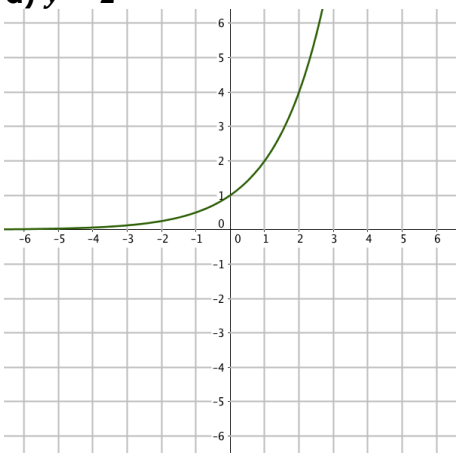
Part 3: Shifts of Exponential and Logarithmic Graphs:

Recall the following basic transformations of functions:

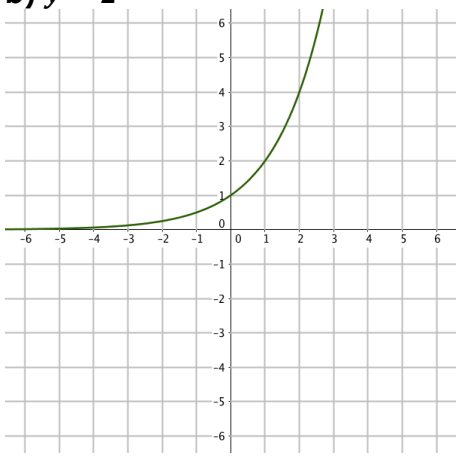
Category 1: $f(x-h)$ horizontal shift (left and right)	**Category 2: $f(x)+k$ vertical shift (up and down)**
Category 3: $f(-x)$ reflection over the y-axis	**Category 4: $-f(x)$ reflection over the x-axis**

1. Using desmos, graph the functions on the same axes as their parent functions:

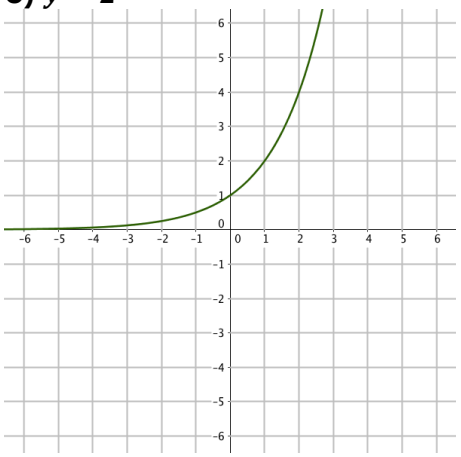
a) $y = 2^{(x+1)}$



b) $y = 2^{(x-2)}$



c) $y = 2^{(x-3)}$

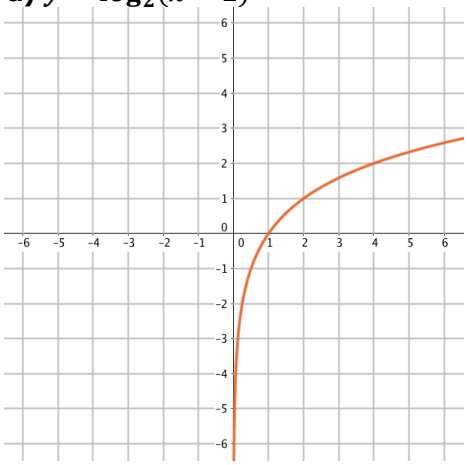


What transformation Category (1,2,3, or 4) do the functions fall under? _____

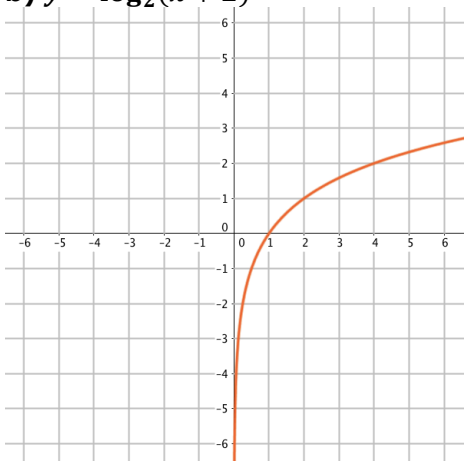
Does the asymptote change in the examples above? Does the x or y intercept change? Explain:

2. Using desmos, graph the functions on the same axes as their parent functions:

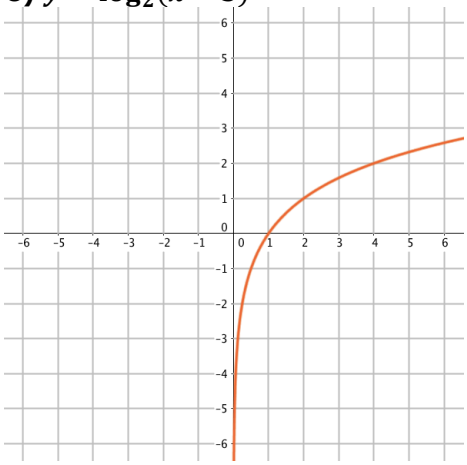
a) $y = \log_2(x-1)$



b) $y = \log_2(x+2)$



c) $y = \log_2(x-3)$

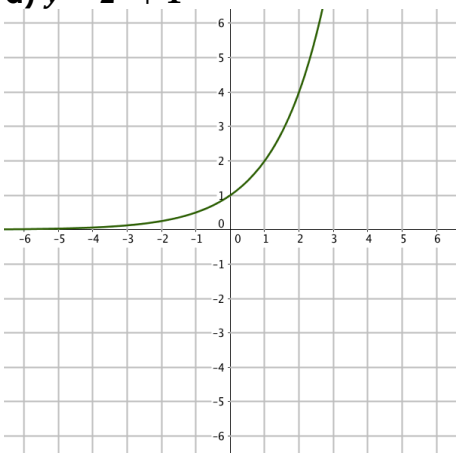


What transformation Category (1,2,3, or 4) do the functions fall under? _____

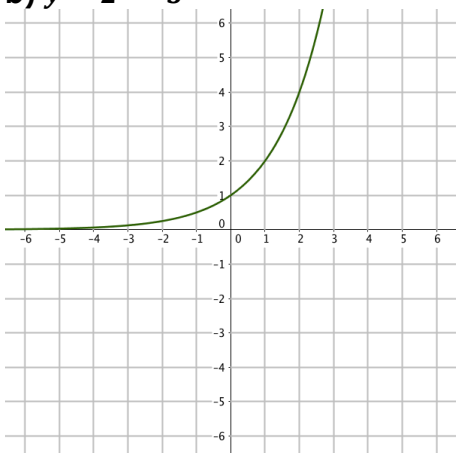
Does the asymptote change in the examples above? Does the x or y intercept change? Explain:

3. Using desmos, graph the functions on the same axes as their parent functions:

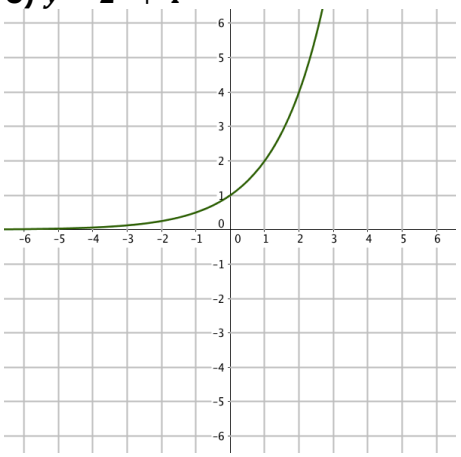
a) $y = 2^x + 1$



b) $y = 2^x - 3$



c) $y = 2^x + 4$

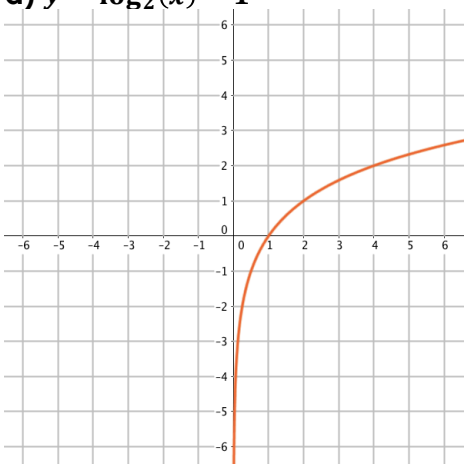


What transformation Category (1,2,3, or 4) do the functions fall under? _____

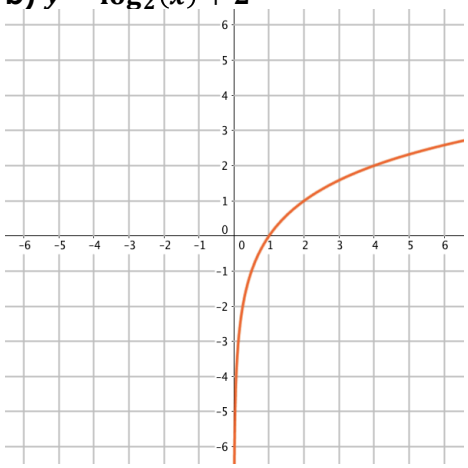
Does the asymptote change in the examples above? Does the x or y intercept change? Explain:

4. Using desmos, graph the functions on the same axes as their parent functions:

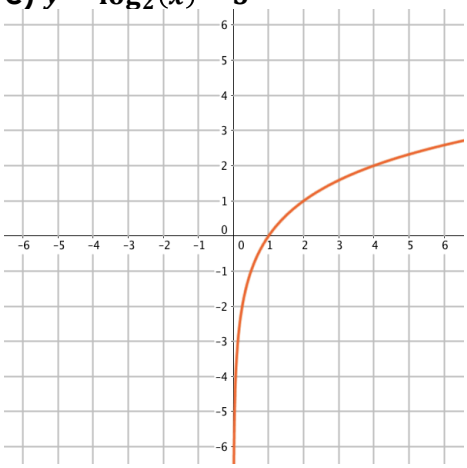
a) $y = \log_2(x) - 1$



b) $y = \log_2(x) + 2$



c) $y = \log_2(x) - 3$



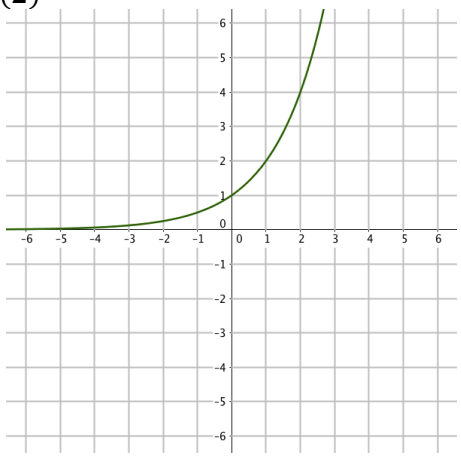
What transformation Category (1,2,3, or 4) do the functions fall under? _____

Does the asymptote change in the examples above? Does the x or y intercept change? Explain:

Part 4: Reflections of Exponential and Logarithmic Graphs:

1. Using desmos, graph the functions on the same axes as their parent functions:

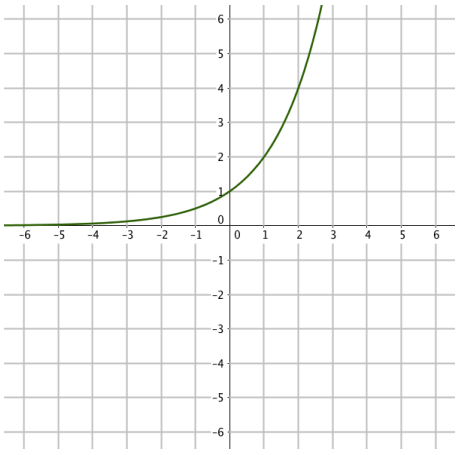
a) $y = -(2)^x$



What transformation Category (1,2,3, or 4) do the functions fall under? _____

Does the asymptote change in the examples above? Does the x or y intercept change? Explain:

b) $y = 2^{-x}$

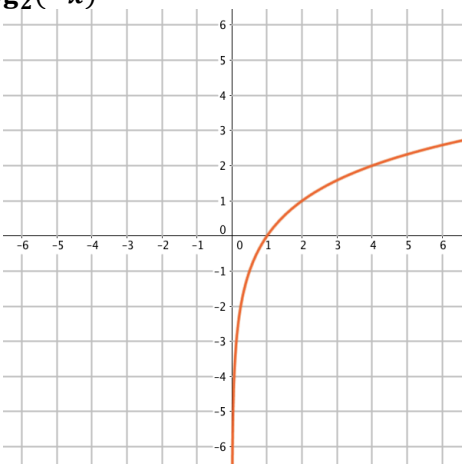


What transformation Category (1,2,3, or 4) do the functions fall under? _____

Does the asymptote change in the examples above? Does the x or y intercept change? Explain:

2. Using desmos, graph the functions on the same axes as their parent functions:

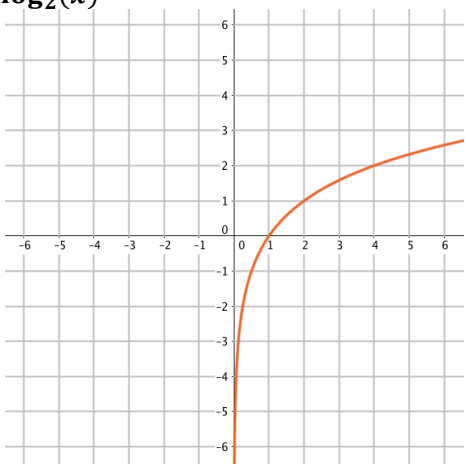
a) $\mathbf{y = \log_2(-x)}$



What transformation Category (1,2,3, or 4) do the functions fall under? _____

Does the asymptote change in the examples above? Does the x or y intercept change? Explain:

b) $\mathbf{y = -\log_2(x)}$



What transformation Category (1,2,3, or 4) do the functions fall under? _____

Does the asymptote change in the examples above? Does the x or y intercept change? Explain:

Part 5: Mastery of Transformations

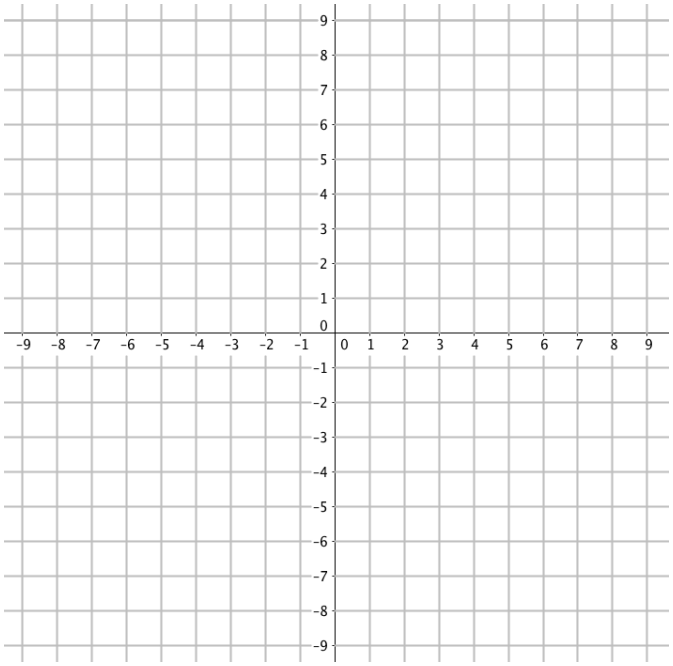
Write function rules for the following transformations:

a) The graph $y = 3^x$ is translated 2 units right	b) The graph $y = 5^x$ is reflected over the x-axis
c) The graph $y = \log_4 x$ is reflected over the y-axis	d) The graph $y = \log_3 x$ is translated 4 units up
e) The graph $y = \log x$ is translated 3 units down and 5 units left.	f) The graph $y = 2^x$ is translated 3 units down and 5 units left.

Graph and label the functions below:

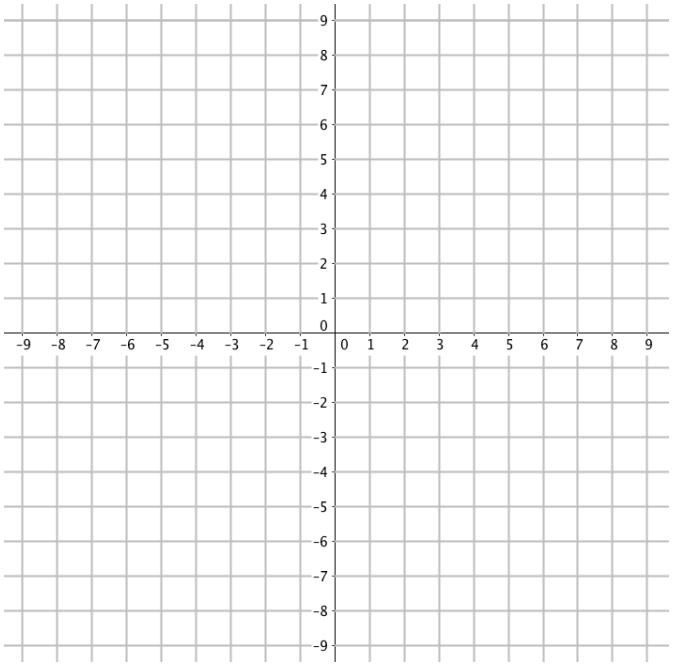
a)

$\boldsymbol{y = 3^x}$
$\boldsymbol{y = 3^{-x}}$
$\boldsymbol{y = 3^x + 2}$
$\boldsymbol{y = 3^{x+4}}$
$\boldsymbol{y = -3^x}$



b)

$\boldsymbol{y = \log_3 x}$ **(hint: how does this relate to** $\boldsymbol{y = 3^x}$**)?**
$\boldsymbol{y = \log_3(-x)}$
$\boldsymbol{y = \log_3(x) + 4}$
$\boldsymbol{y = \log_3(x - 4)}$
$\boldsymbol{y = -\log_3(x)}$



Part 6: Connections to other types of functions:

1. Classify the following functions as Quadratic, Exponential, Radical, or Logarithmic:

a) $y = (x-3)^2 + 2$

Type of Equation:

b) $y = 5^{x-3} + 2$

Type of Equation:

c) $y = \sqrt{x-3} + 2$

Type of Equation:

d) $y = \log(x-3) + 2$

Type of Equation:

a) What do the functions above have in common?

b) What transformation is represented by each of the functions above? How do you know?

2. Below are the parent functions for quadratic, exponential, radical, cubic, absolute value, and logarithmic functions. Write the function rule for a transformation of **6 units left**, **3 units down**. The first one has been done for you:

a) $f(x) = \sqrt{x}$

Example: $f'(x) = \sqrt{x+6} - 3$

b) $f(x) = x^2$

c) $f(x) = 8^x$

d) $f(x) = \log x$

e) $f(x) = x^3$

f) $f(x) = |x|$