Name

AP Calc Practice Set 3
Tangent Lines and Linear Approximation

___ 1. What is the slope of the line tangent to the curve $x^2 + y^2 = 9$ when $x = 3$?

A) -1 B) 0
C) 1 D) π
E) Infinite slope

___ 2. What is the slope of the line tangent to the curve $y = e^x$ at $x = e$.

A) e^π B) 1 C) e D) e^e E) e^x

___ 3. Which of the following is the equation of the line tangent to the curve $f(x) = 2\sin(x)$ at $x = \pi$.

A) $y = -2x + 2\pi$
B) $\frac{y}{2} = -x - \pi$
C) $y = 2x - 2\pi$
D) $\frac{y}{2} = x + \pi$
E) None of the above

___ 4. The tangent line to the curve $y = x^3 - 2x + 4$ at the point $(0, 4)$ has an x–intercept at

A) $(0, 0)$ B) $(-2, 0)$
C) $(2, 0)$ D) $(3, 0)$
E) $(-3, 0)$

___ 5. If $f(x) = 3x^2 - 5$, what is the equation for the tangent line to the curve when $x = 1$?

A) $y + 2 = 6(x - 1)$
B) $y - 2 = -12(x - 1)$
C) $y - 2 = 6(x - 1)$
D) $y - 2 = 6(x + 1)$
E) $y = 6x$

___ 6. If $f(x) = 3x^3 - 5x + 9$, which of the following is the equation for a tangent line to the curve with a slope $m = 4$?

A) $y + 7 = 4(x - 1)$
B) $y + 7 = 4(x + 1)$
C) $y - 11 = 4(x - 1)$
D) $y - 11 = 4(x + 1)$
E) $y - 7 = 4(x + 1)$

___ 7. If $f(x) = 3e^{2x}$, then the equation for the tangent line to the curve parallel to $3y + 3 = 18x$ is

A) $y + 6 = 6x$ B) $y - 6 = 6x$
C) $y - 6 = 3x$ D) $y - 3 = 3x$
E) $y - 3 = 6x$

___ 8. What is the slope of the tangent to the curve $x^4 + x^2y^2 + y^3 = 1$ at the point $(1, -1)$?

A) -6 B) -2 C) $-\frac{6}{5}$ D) $\frac{6}{5}$ E) 2

___ 9. The line $y = 12x + k$ is tangent to the curve $y = x^3$ when k is equal to

A) ± 1 B) ± 8 C) ± 16 D) ± 4 E) 0

___ 10. The approximate value of $y = \sqrt{9 - \sin x}$ at $x = 0.5$, obtained from the tangent to the graph at $x = 0$ is

A) 2.909 B) 2.921
C) 2.917 D) 2.919
E) 2.920

AP Calc Practice Set 3

11. If $f'(10) = 3$ and $f(10) = 6$, the best estimate of $f(10.001)$ using tangent line approximation is

A) 3.003 B) 4.056
C) 5.996 D) 6.003
E) 6.010

12. Use linear approximation on the function $f(x) = \sqrt{x}$ around $x = 4$ to approximate $\sqrt{3}$

A) 1 B) 1.72
C) 1.75 D) 1.8
E) 1.85

13. What is the best approximation for $\sqrt[3]{7.98}$?

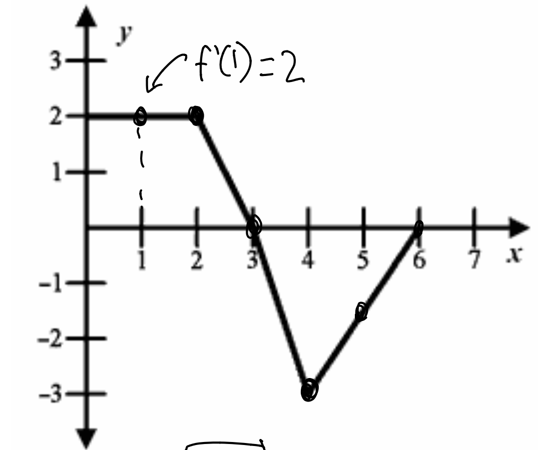
A) 1.890 B) 1.980
C) 1.985 D) 1.989
E) 1.998

14. What is an approximation for $\sqrt{16.01}$?

A) 3.9975 B) 3.99875
C) 4.00125 D) 4.0025
E) 4.05

15. What is the best estimate of $f(3.01)$ if $f(3) = 10$ and $f'(x) = x + 1$?

A) 9.92 B) 9.96
C) 10 D) 10.04
E) 10.08



16. The graph of $\boxed{f'(x)}$ is shown above. If $f(1) = 4$, then the local linearization of f at $x = 1$ is $\boxed{f(x)} =$

A) $2x + 1$ B) $2x + 2$
C) $2x + 3$ D) $2x + 4$
E) $2x - 1$

$f(x) \approx f(1)+f'(1)(x-1)$
$4 + 2(x-1)$
$4 + 2x - 2$
$\boxed{2x+2}$

$f(x) = f(1)+f'(1)$
$f(1.001) = f(1) + f'(1)(1.001-1)$

17. If $f(x)$ is continuous on the interval [0,3] and $f(0) = -3$, $f(3) = 5$ then which of the following must be true?

I. $f(x)$ has a root on the interval.
II. $f(x)$ has a local maxima or minima on the interval.
III. $f'(x)$ is equal to 8/3 at some point on the interval.

A) I only B) II only
C) III only D) I and III
E) I and II

___ 18. If $f(x)$ is continuous and differentiable on a closed interval a to b, and $f(a) = 3$ and $f(b) = 5$, which of the following must be true.

I. The average rate of change on the interval is equal to $\frac{2}{b-a}$.
II. At some point on the interval, the slope of the line tangent to the curve is equal to $\frac{2}{b-a}$.
III. The curve has an inflection point on the interval.

A) I only B) II only
C) III only D) I and II
E) II and III

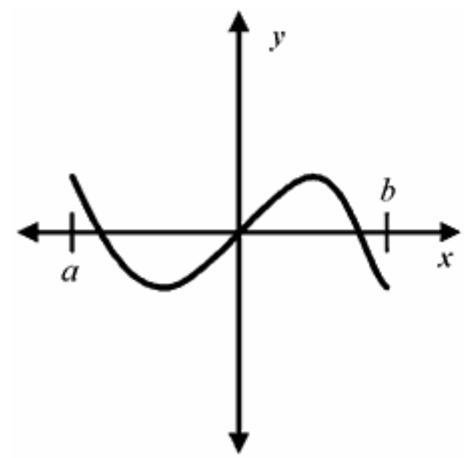
___ 19. What are the value(s) of c that satisfy the Mean Value Theorem for $f(x) = \frac{4}{x} - 2$ on the interval [1, 2]?

A) –2 B) 2 C) $\pm\sqrt{2}$ D) ± 2 E) 0

___ 20. If $f(a) = f(b) = 0$ and $f(x)$ is continuous on $[a, b]$, then

A) $f'(x)$ must be identically zero
B) $f'(x)$ may be different from zero for all x on $[a, b]$
C) there exists at least one number x, $a < c < b$, such that $f'(c) = 0$
D) $f'(x)$ must exist for every x on (a,b)
E) None of the above

___



21. Referring to the graph above, at how many points on the interval $[a, b]$, does the function above satisfy the Mean Value Theorem?

A) 1
B) 2
C) 3
D) 4
E) None of the above

① Slope of tangent line = first derivative

↳ $\boxed{\frac{dy}{dx}}$

$\boxed{(3,0)}$

x y

$x^2+y^2=9$

$3^2+y^2=9$

$9+y^2=9$

$y=0$

$\frac{d}{dx}(x^2+y^2)=\frac{d}{dx}(9)$

$2x+2y\frac{dy}{dx}=0$

$-2x \qquad -2x$

$\frac{2y\frac{dy}{dx}}{2y}=\frac{-2x}{2y}$

$\frac{dy}{dx}=\frac{-x}{y}$

$=\frac{-3}{0}=$ infinite (undefined slope)

(2) Slope $= f'(x)$

$$y = e^x$$
$$y' = e^x$$
$$y' = e^e$$

$\boxed{\text{Ans: D}}$

(3) Equation of tangent line

$$y - y_1 = m(x - x_1)$$

(x_1, y_1) Plug x_1 in original

$(\pi, 0)$

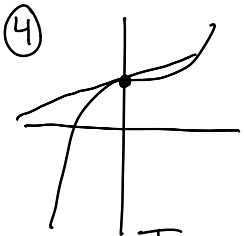
$$f(x) = 2\sin(x)$$
$$f'(x) = 2\cos(x)$$
$$f'(\pi) = 2\cos(\pi)$$
$$= 2(-1) = -2$$

"m" is found by finding the first derivative at that point!

$$y - 0 = -2(x - \pi)$$
$$y = -2x + 2\pi$$

$\boxed{\text{Ans: A}}$

(4)



TANGENT LINE

$y - y_1 = m(x - x_1)$

(x_1, y_1)

$(0, 4)$

Finding m

$y' = 3x^2 - 2$

$y' = 3(0)^2 - 2$

$= -2$

$y - 4 = -2(x - 0)$

$y - 4 = -2x$

$y = -2x + 4$

To find the x-intercept, make $y = 0$

$0 = -2x + 4$

$2x = 4 \quad x = 2$

$(2, 0)$

⑤ $f(x)=3x^2-5$ $\quad x_1=1$

$y-y_1=m(x-x_1)$ $\quad (1,-2)$ — x_1, y_1

$m \downarrow f'(x_1)$

$f'(x)=6x$

$f'(1)=6(1)=6$

$y-(-2)=6(x-1)$ $\quad$ Ans: A

$y+2=6(x-1)$

⑥ $f(x)=3x^3-5x+9$

$y-y_1=m(x-x_1)$

$f'(x_1)=\boxed{4}=m$

$f'(\boxed{x})=9x^2-5$

$f'(x_1)=9x_1^2-5=4$

$+5 \quad +5$

$9x_1^2=9$

$\cancel{x_1=}$ 1 or -1

$x_1=1 \quad y_1=7$

$y-7=4(x-1)$

$x_1=-1 \quad y_1=11$

$y-11=4(x+1)$

Ans: D

8 Implicit! Don't forget Chain rule

slope → $\frac{dy}{dx}$ $x=1$ $y=-1$

$$\frac{d}{dx}\left(x^4+x^2\cdot y^2+y^3\right)=\frac{d}{dx}(1)$$

$u \quad v$

$$4x^3+\underbrace{x^2\cdot 2y\frac{dy}{dx}}_{u\cdot v'}+\underbrace{2x\,y^2}_{u'\cdot v}+3y^2\frac{dy}{dx}=0$$

$-4x^3 \qquad -2xy^2$

Factor out $\frac{dy}{dx}$

$$x^2\cdot 2y\frac{dy}{dx}+3y^2\frac{dy}{dx}=-4x^3-2xy^2$$

$$\frac{\frac{dy}{dx}\cancel{(2x^2y+3y^2)}}{\cancel{(2x^2y+3y^2)}}=\frac{-4x^3-2xy^2}{(2x^2y+3y^2)}$$

Subbing $x=1$ $y=-1$

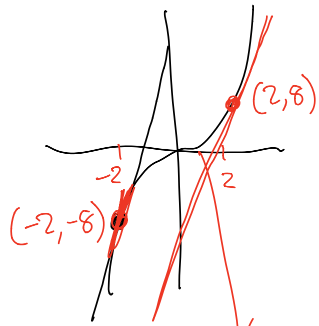
$$\frac{dy}{dx}=\frac{-4x^3-2xy^2}{2x^2y+3y^2}$$

$$=\frac{-4(1)^3-2(1)(-1)^2}{2(1)^2(-1)+3(-1)^2}$$

$$=\frac{-6}{1}=-6$$

Ans: A

9



$y = 12x + k$

slope = 12

when are the slopes the same?

Tangent lines have the same slope as the graph itself (same $f'(x)$)

$y = x^3$ needs a slope of 12

y'

$y' = 3x^2 = 12$

$\frac{3x^2}{3} = \frac{12}{3}$

$x^2 = 4$

$x = \pm 2$

(2, 8)

(-2, -8)

Plug these into $y = 12x + k$ → (2,8): $8 = 12(2) + k$

$8 = 24 + k$

$-24 \quad -24$

$k = -16$

(-2,-8) $-8 = 12(-2) + k$

$-8 = -24 + k$

$+24 \quad +24$

$k = 16$

Ans: C

(10) Linear Approximation

$$f(a+dx) = f(a) + f'(a)\,dx$$

dx ↑ positive or negative

Ex:

$$f(\underbrace{9.01}) = f(9) + f'(9)(.01)$$

(10)

$$f(0.5) = \overset{①}{f(0)} + \overset{②}{f'(0)}\overset{③}{(.5)}$$

using $f(x) = \sqrt{9 - \sin x}$

$$f(0) = \sqrt{9-\sin(0)} = \sqrt{9} = 3$$

$f'(0) =$

$$\hookrightarrow f'(x) = \underset{\text{"Outer"}}{\frac{1}{2\sqrt{9-\sin x}}} \cdot \frac{-\cos(x)}{1} = \frac{-\cos(x)}{2\sqrt{9-\sin x}}$$

$\frac{d}{du}\sqrt{u} = \frac{1}{2\sqrt{u}}$

$$f'(0) = \frac{-\cos(0)}{2\sqrt{9-\sin(0)}} = \frac{-1}{2\sqrt{9}} = \boxed{\frac{-1}{6}}$$

Ans

$$f(0.5) = 3 + \left(\frac{-1}{6}\right)(.5) = 2.917 \quad \text{C}$$

(11) $f(10.001) = f(10) + f'(10)(.001)$ $\quad \Delta x = .001$

$= 6 + \underbrace{(3)(.001)}$

$= 6 + .003$

$= 6.003$ Ans: D

(12) $f(3) = f(4) + f'(4)(-1)$ $\quad 3-4=-1$

approximation

$\approx$

$f'(x) = \frac{1}{2\sqrt{x}}$

$f'(4) = \frac{1}{2\sqrt{4}}$

$= \frac{1}{2\cdot 2} = \frac{1}{4}$

$= 2 + \left(\frac{1}{4}\right)(-1)$

$= 2 - \frac{1}{4}$

$= 1.75$ $\boxed{\text{Ans: C}}$

(13) $f(7.98) = f(8) + f'(8)\Delta x$

$= f(8) + f'(8)(-.02)$

$= 2 + \frac{1}{12}(-.02)$

$= 1.998$

Ans: E

$f(x) = \sqrt[3]{x}$

$f(8) = \sqrt[3]{8}$

$= 2$

$f(x) = x^{1/3}$

$f'(x) = \frac{1}{3}x^{-2/3}$

$= \frac{1}{3x^{2/3}} = \frac{1}{3(8)^{2/3}} = \frac{1}{3(4)} = \frac{1}{12}$

(14) $f(16.01) = f(16) + f'(16)(.01)$ $\quad f(x) = \sqrt{x}$

$= 4 + \frac{1}{8}(.01)$ $\quad f'(x) = \frac{1}{2\sqrt{x}}$

$= 4 + .00125$ $\quad f'(16) = \frac{1}{2\sqrt{16}}$

$= 4.00125$ $\quad = \frac{1}{8}$

Ans: C

(15) $f(3.01 \approx f(3) + f'(3)(.01)$

$10 + 4(.01)$

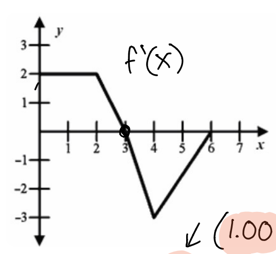
$10 + .04$ $\quad f'(x) = x+1$

10.04 $\quad f'(3) = 3+1$

$=$

$\boxed{\text{Ans: D}}$

(16)



x	f'(x)
0	2
$\boxed{1}$	$\boxed{2}$
2	2
3	0

Ex:

$(1.001 - 1)$

$* \; f(1.001) \approx f(1) + f'(1)\Delta x$

$f(x) = f(1) + f'(1)(x-1)$

$= 4 + (2)(x-1)$

$= 4 + 2x - 2$

$= 2x + 2$ Ans: B

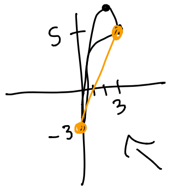
(17)

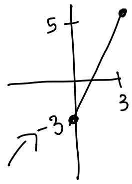
x	f(x)
0	-3
3	5

3   8

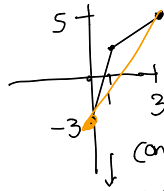
Slope $= \frac{8}{3}$

Average Rate of change (Slope of secant line)





Continuous AND DIFFERENTIABLE



continuous

Not differentiable at $x=1$

1) True

↳ By the Intermediate Value Theorem IVT:

If $f(x)$ is continuous on $[a,b]$, then for every c such that:

$$f(a) < c < f(b)$$

$$f(x) = c \text{ for some } x \text{ on } [a,b]$$

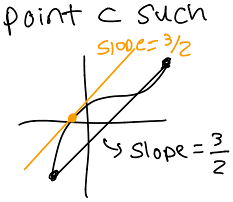
2) False

3) IF this function is DIFFERENTIABLE, then this would be true by the Mean Value Theorem:

⇒ IF a function is continuous & Differentiable on $[a,b]$, then there is a point c such that

$$f'(c) = \frac{f(b)-f(a)}{b-a}$$

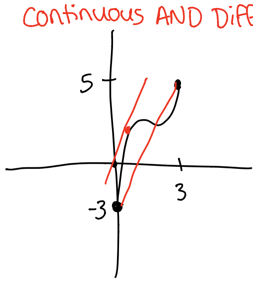
slope of the tangent line = slope of the secant line



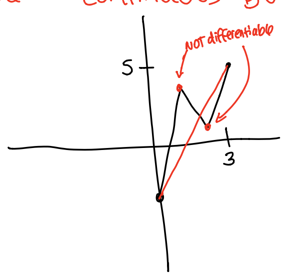
3) is false because $f(x)$ is NOT differentiable.

Ans A

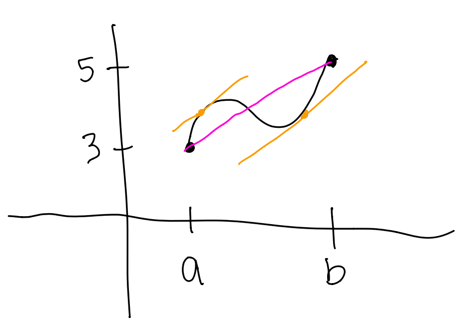
Continuous AND Differentiable



Continuous BUT NOT Differentiable



18)



A.R.O.C
↳ slope of the secant line between $(a, f(a))$ $(b, f(b))$

$b-a\begin{bmatrix} a, & 3 \\ b, & 5 \end{bmatrix}$ $5-3=2$

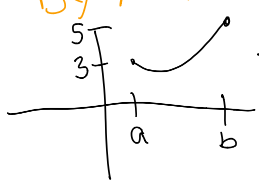
$\frac{2}{b-a}$ = AROC

1) True

2) True because continuous AND DIFFERENTIABILITY

By MVT

3)



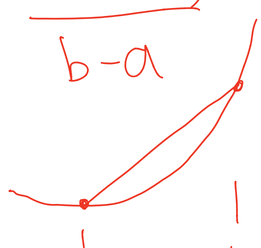
→ 3 is not true!
Example concave up everywhere!

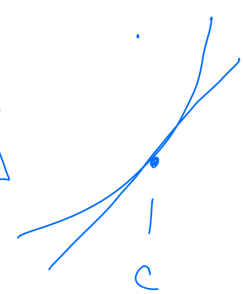
Ans: D

19) Mean Value Theorem

Slope of secant line: $\frac{f(b)-f(a)}{b-a}$ = Slope of tangent line $f'(c)$

↓
Somewhere on $[a,b]$





x	f(x)
1	2
2	0

1 −2

$\boxed{\frac{-2}{1}}$

$f'(c) = \frac{-2}{1}$

$f(x) = \frac{4}{x} - 2 \Rightarrow 4x^{-1}$

A.R.O.C
↓

$f'(x) = -4x^{-2} = \frac{-4}{x^2} = \frac{-2}{1}$

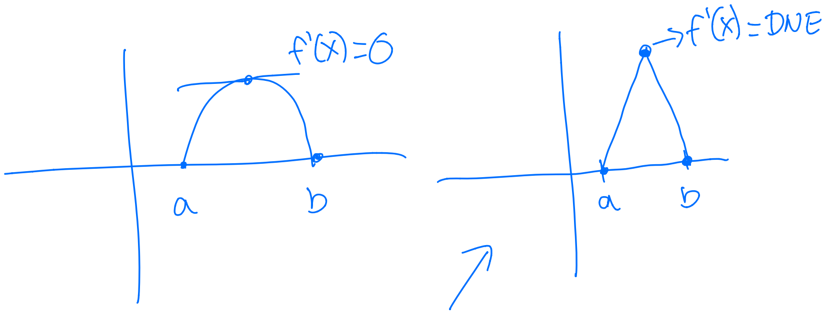
$\frac{-4}{-2} = \frac{-2x^2}{-2}$

$2 = x^2$

$x = \pm\sqrt{2}$

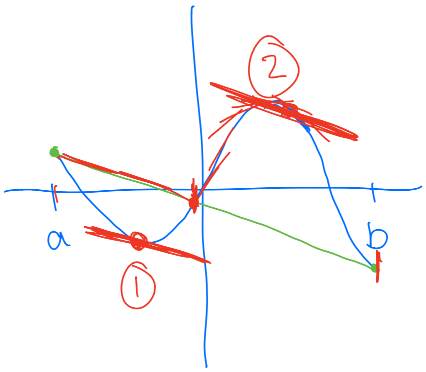
$\boxed{c = \pm\sqrt{2}}$

20



ANS: B)

21



How many points does the secant line have the same slope as the tangent line?

Two poins

Ans : B