Name **AP Calc Practice Set 2**

Limits and Continuity

___ 1. $\lim_{x\to 0} \frac{\sin 3x}{\sin 5x} =$ $\frac{\sin(3(0))}{\sin(5(0))} = \frac{0}{0} \to$ Indeterminant

A) 0
B) $\frac{3}{5}$
C) 1
D) π
E) nonexistent

→ Factor cancel (simplify)
→ L'Hospital's (take the derivative of top & bottom individually and try again)

$\frac{d}{dx}(\sin 3x) = \cos(3x)\cdot 3$

$\frac{d}{dx}(\sin 5x) = \cos(5x)\cdot 5$

$\frac{\cos(0)\cdot 3}{\cos(0)\cdot 5} = \frac{1\cdot 3}{1\cdot 5}$

___ 2. $\lim_{x\to 9} \frac{9-x}{3-\sqrt{x}}$ $\left(\frac{3+\sqrt{x}}{3+\sqrt{x}}\right) = \frac{(9-x)(3+\sqrt{x})}{9+\cancel{3\sqrt{x}}-\cancel{3\sqrt{x}}-x} = \frac{\cancel{(9-x)}(3+\sqrt{x})}{\cancel{(9-x)}}$

↑ radical
→ multiply by the conjugate

$3+\sqrt{x}$

$3+\sqrt{9} =$

$3+3=6$

A) 0
B) 3
C) 6
D) ∞
E) nonexistent

___ 3. $\lim_{x\to 0} \frac{\sin x}{x} =$ $\frac{\frac{d}{dx}\sin(x)}{\frac{d}{dx}(x)} = \frac{\cos(x)}{1} = \frac{\cos(0)}{1} = \frac{1}{1}$

L'Hospital's Rule

A) 0
B) 1
C) 2
D) π
E) nonexistent

___ 4. $\lim_{x\to -3} \frac{x^3+27}{x+3} =$ $\frac{\frac{d}{dx}x^3+27}{\frac{d}{dx}x+3} = \frac{3x^2}{1} = \frac{3(-3)^2}{1} = 27$

A) -3 B) 0 C) 9 D) 18 E) 27

___ 5. $\lim_{x\to 0} \frac{\sqrt{x^2+9}-3}{x} =$

multiply by the conjugate

A) ∞
B) $-\infty$
C) 3
D) 0
E) nonexistent

$$\frac{\overset{a}{\sqrt{x^2+9}}\ \overset{-b}{-3}}{x}\left(\frac{\overset{a}{\sqrt{x^2+9}}\ \overset{+b}{+3}}{\sqrt{x^2+9}+3}\right) = \frac{\overset{a^2}{(x^2+9)}\ \overset{-b^2}{-9}}{x(\sqrt{x^2+9}+3)}$$

$$= \frac{\cancel{x^2}^{\,1}}{\cancel{x}(\sqrt{x^2+9}+3)}$$

$$= \frac{x}{\sqrt{x^2+9}+3} = \frac{0}{6}$$

___ 6. $\lim_{x\to 9} \frac{\sqrt{x}-3}{9-x}$ $\left(\frac{\sqrt{x}+3}{\sqrt{x}+3}\right) = \frac{x-9}{(9-x)(\sqrt{x}+3)}$

$$= \frac{-1\cancel{(9-x)}}{\cancel{(9-x)}(\sqrt{x}-3)}$$

$$= \frac{-1}{\sqrt{x}+3} = \frac{-1}{\sqrt{9}+3}$$

$$= \frac{-1}{3+3}$$

$$= -1/6$$

A) -3
B) $-\frac{1}{6}$
C) $\frac{1}{6}$
D) 3
E) 6

___ 7. $\lim_{h\to 0} \frac{(x+h)^5-x^5}{h} =$

A) $5x^4$
B) x^5
C) 0
D) 5
E) nonexistent

Definition of a derivative

$f'(x) = \lim_{h\to 0} \frac{f(x+h)-f(x)}{h}$

what is f(x)?

$f(x) = x^5$

$f'(x) = 5x^4$

___ 8. $\lim_{h\to 0} \frac{5(h-1)^2+5}{h} =$

A) -10
B) 0
C) 1
D) 5
E) nonexistent

⇒ A derivative evaluated at some point

$f'(a)$

→ what is $f(x)$?
→ what is a?

$a=-1$

$f(x) = 5x^2$

$f'(x) = 10x$

$f'(x)$ at $a=-1$

$f'(-1) = 10(-1)^1$

$= 10(-1) = -10$

$f'(x) = 10x$

$f'(-1) = -10$

___ 9. $\lim_{h\to 0} \frac{8(x+h)^2-8x^2}{h} =$

A) 8
B) $8x$
C) $16x$
D) 0
E) nonexistent

$f(x) = 8x^2$

$f'(x) = 16x$

___ 10. $\lim_{h\to 0} \frac{-3(x+h)^3+3x^3}{h} =$

$-f(x)$ $f(x) = -3x^3$

$f(x) = -3x^3$

$f'(x) = -9x^2$

A) $3x$
B) $-9x^2$
C) 3
D) 0
E) nonexistent

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___ 11. $\lim_{h\to 0} \frac{20(a+h)^2 - 20a^2}{h} =$

A) 20 B) $20a$
C) $40a$ D) 0
E) nonexistent

Definition of Derivative

$f'(x) = \lim_{h\to 0} \frac{f(x+h)-f(x)}{h}$

$f(x) = 20a^2$
$f'(x) = 40a$

___ 12. $\lim_{h\to 0} \frac{\cos(x+h) - \cos x}{h} =$ $-f(x)$

A) $\sin x$ B) $-\sin x$
C) $\cos x$ D) 0
E) nonexistent

$f(x) = \cos(x)$
$f'(x) = -\sin(x)$

___ 13. $\lim_{h\to 0} \frac{-10(x+h)^2 + 10x^2}{h} =$ $-f(x)$

A) $10x$ B) $-20x$
C) 0 D) 5
E) nonexistent

$f(x) = -10x^2$
$f'(x) = -20x$

___ 14. $\lim_{h\to 0} \frac{\overbrace{[a(x+h)^2 + b(x+h) + c]}^{f(x+h)} \overbrace{- (ax^2 + bx + c)}^{-(f(x))}}{h} =$

$f(x) = ax^2+bx+c$
$f'(x) = 2ax+b$

A) $2a+b$ B) $2x+b$ C) $2ax+b$ D) 0 E) nonexistent

___ 15. $\lim_{h\to 0} \frac{\overbrace{\frac{1}{x+h}}^{f(x+h)} - \overbrace{\frac{1}{x}}^{f(x)}}{h} =$

A) $\frac{1}{x}$
B) $\frac{1}{x^2}$
C) $-\frac{1}{x^2}$
D) 0
E) nonexistent

$f(x) = \frac{1}{x} = x^{-1}$
$f'(x) = -1x^{-2}$
$= \frac{-1}{x^2} = -\frac{1}{x^2}$

___ 16. $f'(a) = \lim_{h\to 0} \frac{\overbrace{[5(3+h)]}^{f(x+h)} - \overbrace{15}^{f(x)}}{h} =$

What is $f(x)$?
What is a?

A) 15 B) 5
C) 0 D) -3
E) nonexistent

$f(x) = 5(x)$
$f'(x)$ at 3
$f'(x) = 5$

$\frac{\cancel{15} + 5h - \cancel{15}}{h} = \frac{5\cancel{h}}{\cancel{h}}$

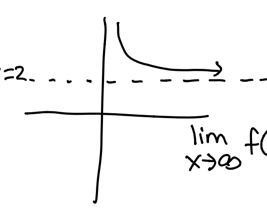
___ 17. $\lim_{h\to 0} \frac{\overbrace{[(x+h)^2 + (x+h)]}^{f(x+h)} - \overbrace{[(x^2+x)]}^{f(x)}}{h} =$

$\boxed{f(x) = x^2+x}$
$f(x+h) =$
$(x+h)^2 + (x+h)$

A) $2x+1$ B) $-x$
C) x^2 D) 0
E) nonexistent

$f'(x) = 2x+1$

___ 18. $\boxed{\lim_{x\to\infty}} \frac{4x^2 + 3x - 3}{5x^2 + 177x + 2} =$



$\lim_{x\to\infty} f(x) = 2$

A) 0 ↳ same highest exponent
B) $\frac{4}{5}$
C) 1
D) $\frac{5}{4}$
E) ∞

Matching the highest exponent

$\frac{x^a}{x^b}$ $a > b$ $\lim_{x\to\infty} = \infty$

$\frac{x^a}{x^b}$ $a < b$ $\lim_{x\to\infty} = 0$

$\frac{c \cdot x^a}{d \cdot x^b}$ $a = b$ $\lim_{x\to\infty} = \frac{c}{d}$

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____ 19. For which pair of functions $f(x)$ and $g(x)$ below, will the $\lim\limits_{x\to\infty} \frac{f(x)}{g(x)} = 0$?

→ grow much larger than f(x)

A) $f(x) = x^{100}$ Polynomial
$g(x) = 2^x$ Exponential

B) $f(x) = e^x$
$g(x) = x^{100}$

C) $f(x) = x^2 + 2x$
$g(x) = x^2$

D) $f(x) = e^{x}$
$g(x) = \ln(x)$

E) $f(x) = x$
$g(x) = \ln(x)$

Exponential Functions grow quicker than Polynomials

Exp: a^x, e^x

Polynomials: x^3, x^{10}, x^{100}

____ 20. $\lim\limits_{z\to\infty} \frac{6z^2 + 2z - 7}{4z^3 + 2z - 7} =$ ← Denominator has larger exponent

$\frac{\#}{\infty}$

A) ∞
B) $\frac{3}{2}$
C) 1
D) $\frac{2}{3}$
E) 0

____ 21. $\lim\limits_{x\to\infty} \frac{\ln x}{x}$ $= \frac{\infty}{\infty}$ → L'Hospitals

A) 0 B) 1 C) π D) e E) ∞

$\frac{\frac{1}{x}}{1} = \frac{1}{x}$

$\lim\limits_{x\to\infty} \frac{1}{x} = \frac{1}{\infty} = 0$

____ 22. $\lim\limits_{x\to\infty} \frac{2x^1 - 4}{3x^1 + 5} =$

A) $-\frac{4}{5}$
B) 0
C) $\frac{2}{3}$
D) 1
E) ∞

____ 23. $\lim\limits_{x\to\infty} \frac{\sqrt{3x^2 + 5}}{x + 2} =$

$\frac{\sqrt{3x^2}}{x} = \frac{\sqrt{3}\sqrt{x^2}}{x} = \frac{\sqrt{3}\,\cancel{x^1}}{1\,\cancel{x^1}}$

A) $\frac{\sqrt{5}}{2}$
B) $\frac{5}{2}$
C) 3
D) $\sqrt{3}$
E) ∞

____ 24. $\lim\limits_{x\to\infty} \frac{2 - \boxed{3x^2}}{\boxed{2x^2} - 4x} =$ $\frac{-3}{2}$

A) $-\frac{3}{2}$
B) $\frac{1}{2}$
C) $\frac{3}{4}$
D) 1
E) ∞

____ 25. $\lim\limits_{x\to-\infty} \frac{x^1 - 3}{x^2 + 4x + 2} =$

→ Becomes bigger faster (higher degree)

A) ∞
B) $\frac{1}{4}$
C) 0
D) $-\frac{3}{2}$
E) $-\infty$

____ 26. $\lim\limits_{x\to\infty} \frac{3x^7 - 2x}{4x^6 + 1} =$ → higher degree

A) $-\infty$
B) -2
C) 0
D) $\frac{3}{4}$
E) ∞

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____ 27. $\lim_{x\to\infty} \frac{3x^2+1}{(3-x)(3+x)} =$ $\frac{3x^2+1}{9-x^2} = \frac{3}{-1}$

A) -9 B) -3 C) 1 D) 3 E) 9

____ 28. The function $f(x) = \begin{cases} 9-x^2, x<2 \\ ax+b, x\geq 2 \end{cases}$ is continuous and differentiable for all values of x. Find $a$and b.

A) $a=-2, b=9$ B) $a=-5, b=15$
C) $a=0, b=5$ D) $a=3, b=2$
E) $a=-4, b=13$

____ 29. For what value of c and k is the function $f(x) = \begin{cases} 2x+c, x\leq -1 \\ kx^2 \quad, x>-1 \end{cases}$ both differentiable and continuous for all real values of x.

A) $c=-1, k=1$ B) $c=1, k=-1$
C) $c=1, k=1$ D) $c=2, k=-2$
E) $c=-2, k=-2$

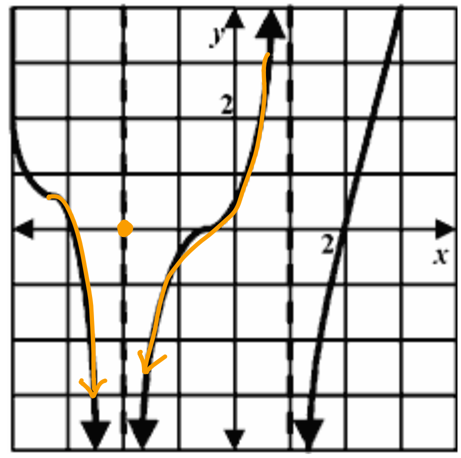
____ 30. Let $f(x) = \begin{cases} x^3+a-2, x>2 \\ ax^2 \quad, x\leq 2 \end{cases}$. Find the value of a so that $f(x)$ is continuous for all real x.

A) 1 B) 2 C) $\frac{5}{2}$ D) 3 E) 4

____ 31. If $f(x) = \begin{cases} e^x+3, x>0 \\ ax+b, x\leq 0 \end{cases}$ is both continuous and differentiable at $x=0$, then $a+b=$

A) 3 B) 4 C) 5 D) 6 E) 7

____ 32. Base your answer to the following question on "the graph below of $f(x)$.



"

$\lim_{\boxed{x\to -2}} f(x) =$ → If $\lim_{x\to a} f(x) = \infty$ or $-\infty$

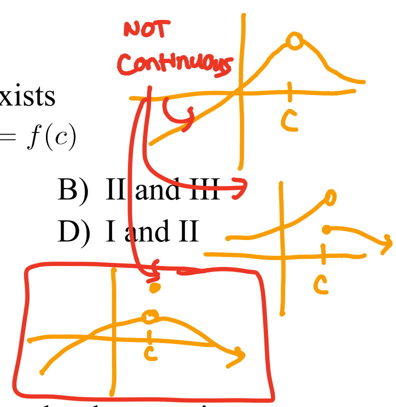
A) $-\infty$ B) 0
C) 1 D) ∞
E) nonexistent

vertical Asymptote at x=8

____ 33. In order for a function to be continuous at point $x=c$, which of the following must be true?

I. $f(c)$ exists
II. $\lim_{x\to c} f(x)$ exists
III. $\lim_{x\to c} f(x) = f(c)$

A) I B) II and III
C) III D) I and II
E) I, II, and III



____ 34. A function is given by the equation

$$f(x) = \begin{cases} x+1 \quad, x<2 \\ x^2 \quad\quad, x=2 \\ 2x-1, x>2 \end{cases}$$

f(2) exist?
$\lim_{x\to 2} f(x)$ exist?
$f(2) = \lim_{x\to 2}$?

Is $f(x)$ continuous at $x=2$?

A) Yes, because $\lim_{x\to 2} f(x) = 3$.
B) Yes, because $\lim_{x\to 2^-} f(x) = 3$. $f(2)=2^2=4$
C) Yes, because $\lim_{x\to 2^+} f(x) = 3$. $\lim_{x\to 2^-} = 3$
D) Yes, because $f(2)=4$.
E) No, because $\lim_{x\to 2} f(x) \neq f(2)$ $\lim_{x\to 2^+} = 3$

$f(2) \neq \boxed{\lim_{x\to 2} = 3}$

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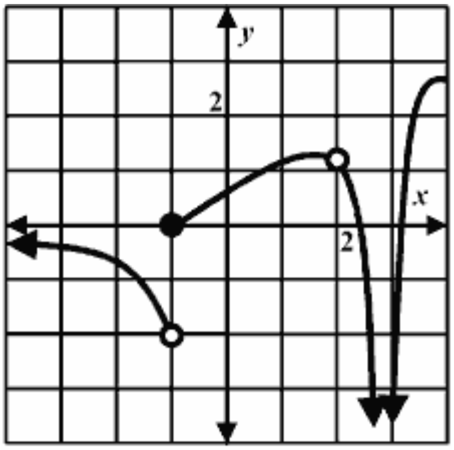
____ 35. Base your answer to the following question on "the graph of $f(x)$ below.



"

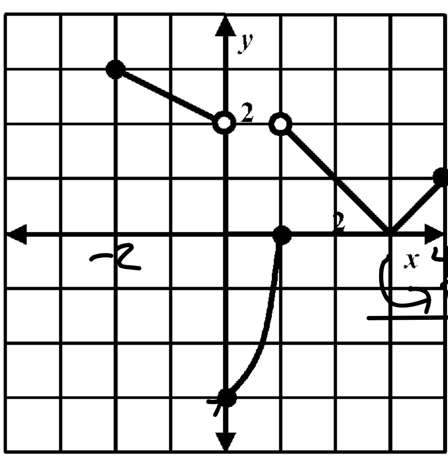
$\lim_{x \to -1} f(x) =$

$\lim_{x \to -1^-} f(x) = -2$

$\lim_{x \to -1^+} f(x) = 0$

A) ∞ B) $\frac{3}{2}$
C) 0 D) $-\infty$
E) nonexistent

Base your answers to questions **36** through **39** on the graph below.



Not
Differentiable
(sharp corner)

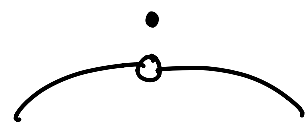
____ 36. The function has a jump discontinuity at

A) $x = 1$
B) $x = -1$
C) $x = 3$
D) $x = 4$
E) None of the above

____ 37. Over which of the following intervals is the function continuous?

A) $0 \le x \le 1$
B) $-1 \le x \le 1$
C) $1 \le x \le 3$ not including
D) $0 \le x \le 4$
E) None of the above

____ 38. The function has a removable discontinuity at



A) $x = 1$
B) $x = 2$
C) $x = 3$
D) $x = 0$
E) None of the above

____ 39. The function is defined on [–2,4] which

↳ x values have a y-value

A) if $x \ne 1$
B) if $x \ne 2$
C) if $x \ne 3$
D) if $x \ne 0$
E) for all x in the interval

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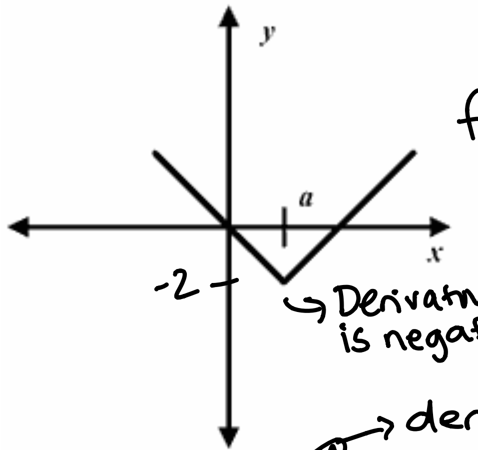
___



$f'(x)$

-2

↳ Derivative is negative

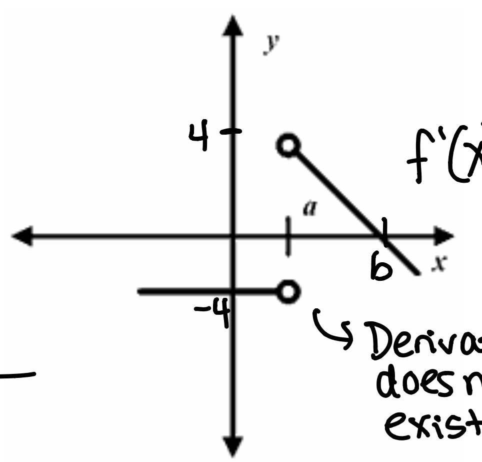
→ derivative

40. The above is a graph of f'. Which of the following statements is true of f at $x = a$?

I. f is continuous
II. f is differentiable
III. f is decreasing

A) I   B) I, II, and III
C) III   D) I and III
E) II and III

___



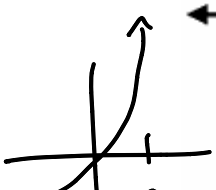
$f'(x)$

4

-4

b

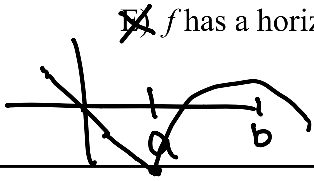
↳ Derivative does not exist at a



a

41. The above is a graph of f'. Which of the following statements is always false?

A) f has a vertical asymptote at $x = a$
B) f is continuous at $x = a$
C) f has a jump discontinuity at $x = a$
D) f has a removable discontinuity at $x = a$
E) f has a horizontal asymptote at $x = a$



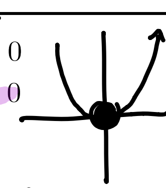
a   b

↳ $\lim_{x\to\infty}$

___ 42. The function $f(x) = \begin{cases} \frac{x^3}{x}, & x \neq 0 \\ 0, & x = 0 \end{cases}$

$\to x^2$

$1 \to$



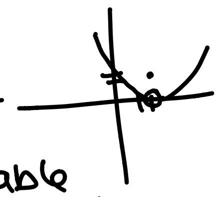
A) is continuous everywhere
B) is continuous except at $x = 0$
C) has a removable discontinuity at $x = 0$
D) has an infinite discontinuity at $x = 0$
E) has a vertical asymptote at $x = 0$

___ $f(x) \begin{cases} \frac{x^2-4}{x-4}, & x \neq -2, 2 \\ \boxed{2, \quad x = 2} \end{cases}$

$\lim_{x\to 2} \quad \frac{2^2-4}{2-4} = \frac{0}{-2} = 0$

43. Which of the following statements is/are true?

I. $\lim_{x\to 2} f(x)$ exists
II. $f(2)$ exists
III. f is continuous at $x = 2$



A) I only
B) II only
C) I and II
D) I, II, and III
E) None of the above

Removable discontinuity

$\lim_{x\to 0} f(x) = \frac{-1}{3}$ $\frac{\cancel{x}(x-1)}{3\cancel{x}}$ $\frac{0}{0}$

___ 44. What value of k makes $f(x) = \begin{cases} \frac{x^2-x}{3x}, & x \neq 0 \\ k, & x = 0 \end{cases}$ continuous at $x = 0$?

$\uparrow -1/3$

A) -1 B) $-\frac{1}{3}$ C) 0 D) $\frac{1}{3}$ E) 1

$f(0) = \lim_{x\to 0} f(x) = -1/3$

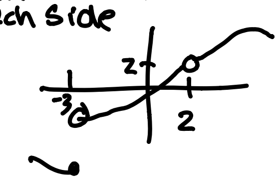
-1/3

___ 45. If $\lim_{x\to 2^-} f(x) = 2$, $\lim_{x\to 2^+} f(x) = 2$, and $f(2)$ is undefined. Which of the following statements are true?

I. $\lim_{x\to 2} f(x) = 2$
II. f is continuous everywhere except at $x = 2$
III. f has a removable discontinuity at $x = 2$

A) I
B) I and III
C) III
D) I, II, and III
E) None of the above

limits are equal from each side



2   -3   2

$$f(x) = \begin{cases} \frac{x^3+x^2}{x^2}, & x < 0 \\ \boxed{1, \quad x = 0} \end{cases}$$

$$\frac{\cancel{x^2}(x+1)}{\cancel{x^2}}$$

46. Which of the following statements is/are true

I. $f(0)$ exists

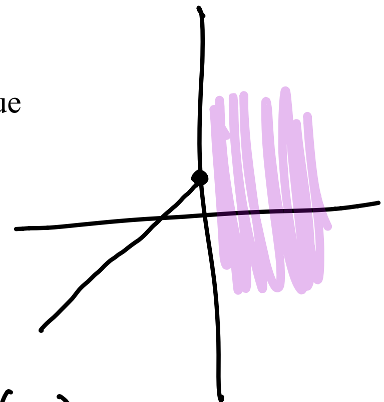
II. $\lim_{x \to 0} f(x)$ exists

III. f is continuous at $x = 0$

A) I B) II

C) III D) I and II

E) I, II, and III



$$\lim_{x \to 0} (x+1) = 1$$

28 Continuous:

$$\lim_{x\to 2^-} f(x) = \lim_{x\to 2^+}$$

$$9-(2)^2 = a(2)+b$$

$$\boxed{5 = 2a+b}$$

$$5 = 2(-4)+b$$

$$5 = -8+b$$

$$b = 13$$

Differentiable:

$$\lim_{x\to 2^+} f'(x) = \lim_{x\to 2^-} f'(x)$$

$$-2x = a$$

$$-2(2) = a$$

$$\boxed{-4 = a}$$

Answer: E

29

Continuous:

$$2x + c = kx^2$$

$$2(-1) + c = k(-1)^2$$

$$\boxed{-2 + c = k}$$

$$-2 + c = -1$$

$$+2 \qquad +2$$

$$\boxed{c = 1}$$

Differentiable:

$$2 = 2kx$$

$$2 = 2k(-1)$$

$$\frac{2}{-2} = \frac{-2k}{-2}$$

$$\boxed{-1 = k}$$

Ans: B

30

$$\lim_{x \to 2^-} f(x) = \lim_{x \to 2^+} f(x)$$

$$x^3 + a - 2 = ax^2$$

$$2^3 + a - 2 = a(2)^2$$

$$\underbrace{8 + a - 2} = 4a$$

$$6 + a = 4a$$

$$-a \quad -a$$

$$6 = 3a$$

$$\frac{6}{3} = \frac{3a}{3}$$

$$a = 2$$

Ans: B

31

Continuous:

$$e^x + 3 = ax + \underbrace{b}$$
$$-ax \quad -ax$$

$$e^x + 3 - ax = \underbrace{b}$$

Differentiable

$$e^x = \underbrace{a}$$

$$e^x + e^x + 3 - ax =$$
$$e^0 + e^0 + 3 - 0 =$$
$$1 + 1 + 3 = 5$$

$\boxed{\text{Ans: } C}$