Name **AP Calc Practice Set 1**

Derivatives

____ 1. If $f(x) = 2x \cdot e^{x^2}$, find $f'(x)$.

A) $(4x)e^{x^2}$ B) $(4x^2+2)e^{x^2}$
C) $(4x^2)e^{x^2}$ D) $(4x^2+2)x^2e^{x^2-1}$
E) $(4x+2)e^{x^2}$

____ 2. If $g'(x) = f(x)$ and $f'(x) = g(2x)$, then $\frac{d^2}{dx^2}(g(2x^2)) =$

A) $4f(2x^2) + 8x^2g(2x^2)$
B) $4f(2x^2) + 8x^2g(4x^2)$
C) $4f(4x^2) + 8x^2g(2x^2)$
D) $4f(2x^2) + g(4x^2)$
E) $g(4x^2)$

Skip

____

x	$f(x^2)$	$g(x^2)$	$f'(x^2)$	$g'(x^2)$
0	-3	0	2	5
1	-1	-3	2	4
2	6	2	-2	0

3. Using the table above find $\frac{d}{dx}\left[f(x^2)g(x^2)\right]$ at $x = 1$.

A) 6 B) 8 C) -4 D) -10 E) -20

____ 4. $\frac{d}{dx}\left(\sqrt{e^x}\right) =$

A) $\frac{e^x}{2\sqrt{e^x}}$
B) $\frac{e^{\sqrt{x}}}{2\sqrt{x}}$
C) $\frac{e^x}{\sqrt{x}}$
D) $\frac{2\sqrt{x}}{e^x}$
E) None of the above

____ 5. If $f(x) = \ln(\sin(3x))$ find $f'(x)$.

A) $3\tan(3x)$ B) $\cot(3x)$
C) $3\csc(3x)$ D) $\tan(3x)$
E) $3\cot(3x)$

____ 6. Find $\frac{dy}{dx}$ if $x^5y + xy^2 = 4$

A) $\frac{(5x+y)}{(x^5+2xy)}$
B) $\frac{y(5x+y)}{(x^5+2x)}$
C) $\frac{-y(5x^4+y)}{(x^5+2xy)}$
D) $\frac{-y(5x+y)}{(x^4+2)}$
E) $5x^4y + y^2$

____ 7. If $x\sin y + y\sin x = 0$, then $\frac{dy}{dx}$ at $\left(\frac{\pi}{4}, \frac{\pi}{4}\right) =$

A) -1 B) 1 C) 0 D) $\frac{\pi}{4}$ E) $-\frac{\pi}{4}$

____ 8. If $25 = x^2 + y^2$ what is the slope of the tangent line when $x = 5$?

A) -1 B) 0
C) 1 D) $\sqrt{2}$
E) Infinite slope

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___ 9. Base your answer to the following question on the following table.

x	f	f'	g	g'
0	1	2	10	-6
1	2	3	3	-4
2	4	5	1	-2
3	9	10	0	0

If $g(x) = f^{-1}(x)$, then $g'(2) =$

A) 1
B) 2
C) 3
D) 4
E) $\frac{1}{2}$

Skip

___ 10. If $f(x) = x^3 - 2x^2 + 4x - 3$ and $g(x) = f^{-1}(x)$, then $g'(-3) =$

A) $\frac{1}{43}$
B) $\frac{1}{4}$
C) 1
D) 4
E) 43

Skip

___ 11. $\lim_{x \to \frac{\pi}{2}} \frac{4\sin(2x - \pi)}{2x - \pi} =$

A) 0 B) 1
C) 2 D) 4
E) nonexistent

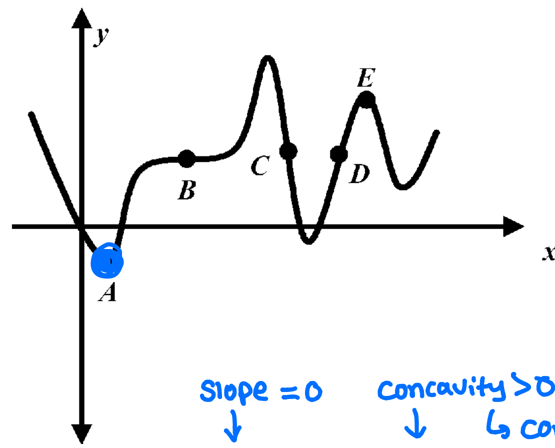
___ 12. $\lim_{x \to 0} \frac{\sin x - x}{x^3} =$

A) -1
B) $-\frac{1}{4}$
C) $-\frac{1}{6}$
D) 0
E) nonexistent

___ 13. $\lim_{x \to 0} \frac{e^{4x} - e^{2x}}{x} =$

A) $-\infty$ B) 0 C) 2 D) 4 E) ∞

___ 14. Base your answer to the following question on the graph below of $f(x)$.



slope = 0 concavity > 0 ↳ concave up

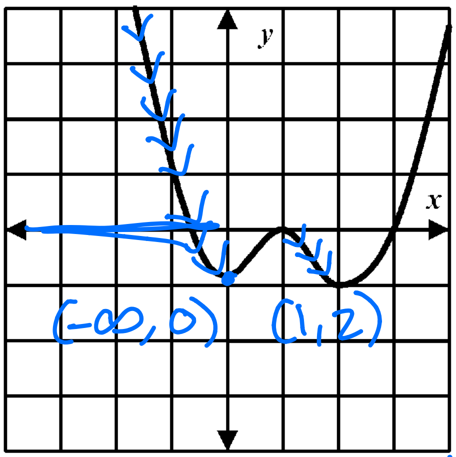
At which point is $f'(x) = 0$ and $f''(x) > 0$?

A) A B) B C) C D) D E) E

___ 15. $\lim_{x \to \infty} \frac{\ln x}{x^2} =$

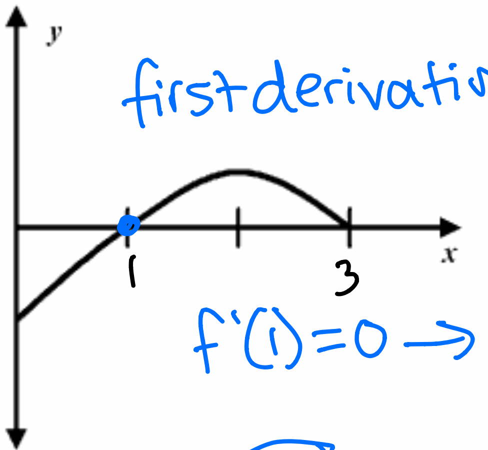
A) e B) $\ln(2)$
C) 0 D) 1
E) ∞

____ 16. Base your answer to the following question on the graph below of $f(x)$.



Where is $f'(x) < 0$? → Slope of tangent line negative?

A) $(0, 1)$ and $(2, \infty)$
B) $(-\frac{1}{2}, 3)$
C) $(\infty, 0)$ and $(1, 2)$
D) $(-\infty, 0)$ and $(1, 2)$
E) $(-\infty, -\frac{1}{2})$ and $(\frac{3}{2}, \infty)$



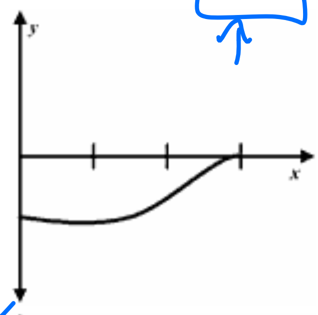
first derivative

x	1	3
f'(x)	− 0 +	0

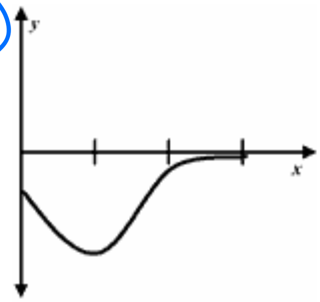
f'(1)=0 → slope of the tangent line of f(x)=0 at x=1

17. Given the graph of $\boxed{f'(x)}$ above, which of the following could be the graph of $f(x)$?

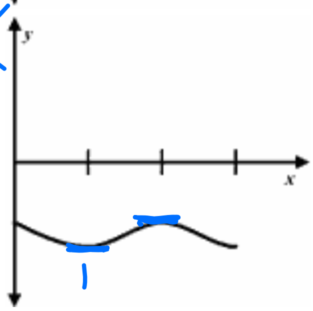
A)



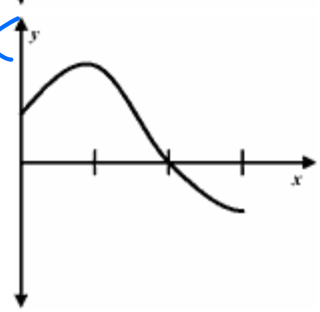
B)



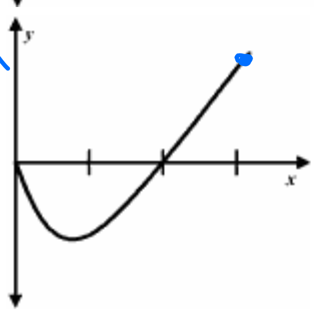
~~C)~~



~~D)~~



~~E)~~



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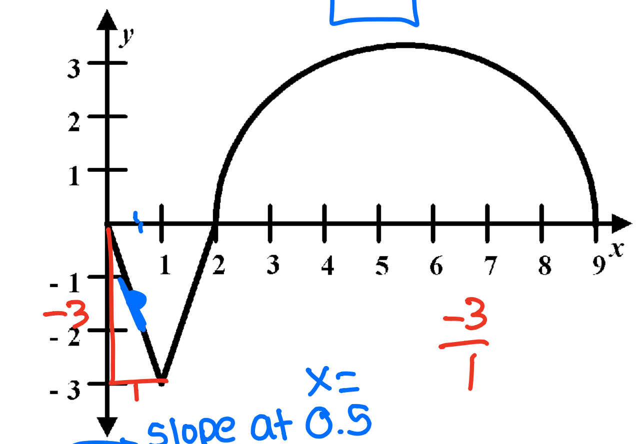
___ 18. Base your answer to the following question on the graph below of $\boxed{f(x)}$.



$f'(0.5) =$

A) $\frac{1}{2}$ $\quad$ $\boxed{\text{B) } -3}$ $\quad$ C) 3 $\quad$ D) $\sqrt{3}$ $\quad$ E) $-\sqrt{3}$

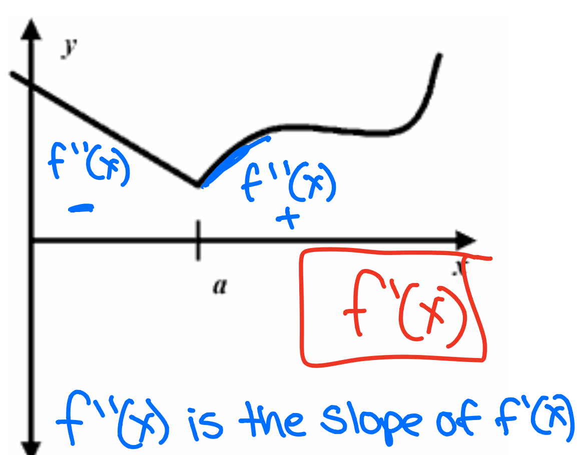
___ 19. $\lim_{h\to 0} \dfrac{(x+h)^3 + 3(x+h) + 2 - (x^3 + 3x + 2)}{h} =$

A) x^3 $\quad$ B) $-3x - 2$

C) $3x^2 + 3$ $\quad$ D) 0

E) nonexistent

___



20. The above is a graph of $\boxed{f'(x)}$. Which of the following is true about f?

Original

A) f is not continuous at $x = a$

B) f has a relative maximum at $x = a$

C) f has a relative minimum at $x = a$

D) f has an inflection point at $x = a$

E) f is not differentiable at $x = a$

f'(x) is not differentiable

f' goes from ↗ ↘

\+ 0 −

f'' changes from − to +

\+ to −

___ 21. Find $\dfrac{d}{dx}\left(\dfrac{1+\cos 2x}{2}\right)$.

A) $\sin 2x$ $\quad$ B) $-\sin x \cos x$

C) $-\cos 2x$ $\quad$ D) $-2\cos x \sin x$

E) $-2\cos 2x \sin x$

___ 22. If $f(x) = 4\cot x + 3\tan x$, then $f'\left(\frac{\pi}{4}\right) =$

A) –2 $\quad$ B) 2 $\quad$ C) –6 $\quad$ D) 6 $\quad$ E) 8

___ 23. If $f(x) = -4x^2 \tan x$, then $f'\left(\dfrac{\pi}{4}\right) =$

A) $-\dfrac{\pi(\pi+4)}{2}$

B) π^2

C) 0

D) $\dfrac{\pi(\pi+4)}{2}$

E) $-\pi^2$

___ 24. $\dfrac{d}{dx}(\csc x) =$

A) $-\csc x \cot x$ $\quad$ B) $-\csc x \tan x$

C) $\csc x \cot x$ $\quad$ D) $-\csc x \tan x$

E) $\sec x \cot x$

___ 25. If $f(x) = 2\sin x - 3\cos x$, then $f'(\pi) =$

A) 1 $\quad$ B) 2 $\quad$ C) 3 $\quad$ D) –2 $\quad$ E) –3

___ 26. If $f(x) = (x\sqrt{x} - x)(x^2 + x)$, $f'(1) =$

A) 2 $\quad$ B) 1 $\quad$ C) 0 $\quad$ D) –1 $\quad$ E) –2

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____ 27. If $f(x) = \sqrt{\dfrac{x^3-1}{x^3+1}}$, then $f'(2) =$

A) $\dfrac{\sqrt{7}}{3}$
B) $\dfrac{3}{2\sqrt{7}}$
C) $\dfrac{4}{9\sqrt{7}}$
D) $\dfrac{2\sqrt{7}}{3}$
E) $\sqrt{\dfrac{11}{13}}$

____ 28. If $y = e^{-x}\sin 2x$, $\dfrac{dy}{dx} =$

A) $-e^{-x}(2\sin 2x + \cos 2x)$
B) $-e^{-x}(\cos 2x - \sin 2x)$
C) $2e^{-x}\cos 2x$
D) $-e^{-x}(\sin 2x + \cos 2x)$
E) $-e^{-x}(\cos 2x)$

____ 29. If $f(x) = 2e^x - 3^x$, then $f'(0) =$

A) -1
B) 5
C) $2 - \ln 3$
D) $2 - 3\ln 3$
E) None of the above

____ 30. If $y = \dfrac{e^x - e^{-x}}{e^x + e^{-x}}$, then $y' =$

A) 1
B) $\dfrac{1}{e^{2x} + e^{-2x}}$
C) $\dfrac{2}{(e^x + e^{-x})^2}$
D) $\dfrac{4}{(e^x + e^{-x})^2}$
E) 0

____ 31. The position of a particle at time t is given by $s(t) = e^{t^2}$. What is the velocity of the particle at $t = 1$?

A) $2e$ B) 1 C) e D) 0 E) $2e^2$

____ 32. For what value of c and k is the function $f(x) = \begin{cases} 2x + c, x \le -1 \\ kx^2 \quad, x > -1 \end{cases}$ both differentiable and continuous for all real values of x.

A) $c = -1, k = 1$ B) $c = 1, k = -1$
C) $c = 1, k = 1$ D) $c = 2, k = -2$
E) $c = -2, k = -2$

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____ 33. Let f be a differentiable function, defined for all real numbers x, with the following properties.

I. $f'(x) = ax^3 + bx^2$
II. $f'(1) = -1$ and $f''(1) = 6$
III. $\int_0^1 f(x) = \dfrac{13}{20}$

Find $f(x)$. Show your work.

____ 34. The depth, in feet, of the water in a pond is a differentiable function D of position x. The table below shows the depth of the water recorded at every 2 meters.

x(feet)	0	2	4	6	8	10
$D(x)$(meters)	10	28	32	30	26	22

(a) Use data from the table to find an approximation for $D'(6)$. Show all work and include units of measure.
(b) Approximate the average depth, in feet, of the water over the interval $0 < x < 10$ meters by using a trapezoidal approximation with subintervals of length $\Delta x = 2$.
(c) A student proposes that $f(x) = 10 + 15xe^{-0.25x}$ is a model for the depth of the water as a function of position, where x is measured in meters and $f(x)$ in feet. Find $f'(6)$.
(d) Use the function f as defined in part (c) to find the average value, in feet, of $f(x)$ over the interval $0 < x < 10$.

____ 35. Let f be a function defined by

$$f(x) = \begin{cases} -x^3 + 3x & \text{for } x \leq 2 \\ 2x^2 + kx + p & \text{for } x > 2 \end{cases}$$

(a) For what values of k and p will f be continuous and differentiable at $x = 2$?
(b) For the values of k and p found in part (a), on what interval(s) is f increasing?
(c) Evaluate $\int_1^3 f(x)dx$

① Product Rule

$u = 2x$ $\quad u' = 2$

$v = e^{x^2}$ $\quad v' = 2x \cdot e^{x^2}$ Chain Rule

$u \cdot v' + u' \cdot v$

$2x \cdot (2x \cdot e^{x^2}) + 2(e^{x^2})$

$4x^2 e^{x^2} + 2e^{x^2}$

$e^{x^2}(4x^2+2)$

ANS: B

② Second derivative

Take derivative of $g(2x^2)$ twice

$\frac{d}{dx} g(2x^2) = g'(\underline{2x^2}) \cdot 4x$

$f(2x^2) \cdot 4x$

$\frac{d}{dx} f(2x^2) \cdot 4x$

$\quad u \cdot v$

Product & Chain Rule

$u = f(2x^2)$

$u' = f'(2x^2) \cdot 4x$

$v = 4x$

$v' = 4$

$u \cdot v' + u' \cdot v$

$f(2x^2) \cdot 4 + f'(2x^2) \cdot 4x \cdot 4x$

$4f(2x^2) + 16x^2 f'(2x^2)$

$g(4x^2)$

(3)

$\frac{d}{dx}\left[f(x^2) \cdot g(x^2)\right]$

$u \quad v$

$u = f(x^2)$

$u' = f'(x^2) \cdot 2x$

$v = g(x^2)$

$v' = g'(x^2) \cdot 2x$

$u' \cdot v + v' \cdot u$

$f'(x^2) \cdot 2x \cdot g(x^2) + g'(x^2) \cdot 2x \cdot f(x^2)$

$f'(1^2) \cdot 2(1) \cdot g(1^2) + g'(1^2) \cdot 2(1) \cdot f(1^2)$

$[f'(1) \cdot 2 \cdot g(1)] + [g'(1) \cdot 2 \cdot f(1)]$

$[2 \cdot 2 \cdot (-3)] + [4 \cdot 2 \cdot (-1)]$

$[-12] + [-8]$

-20

(4) $\frac{d}{dx}(\sqrt{e^x})$

$u^{-1/2} = \frac{1}{u^{1/2}}$

$u = e^x$

$\frac{1}{\sqrt{u}}$

$\frac{d}{dx}(\sqrt{u})$

$\downarrow$ derivative $u^{1/2}$

$\downarrow$ derivative e^x

$\frac{1}{2}u^{-1/2} \cdot e^x$

$\frac{1}{2\sqrt{u}} \cdot \frac{e^x}{1} = \frac{1}{2\sqrt{e^x}} \cdot \frac{e^x}{1}$

$\boxed{\text{Ans: A} : \frac{e^x}{2\sqrt{e^x}}}$

⑤ Three function

$f'(x) \rightarrow$ chain rule 3 times

Outer: $\ln(u)$ ✓ $u = \sin(3x)$

Derivative $\frac{1}{u}$

inner: $\sin(v)$ $v = 3x$

Derivative $\cos(v)$

Inner-Inner $(3x)$

Derivative $= 3$

$$\frac{1}{\sin(3x)} \cdot \cos(3x) \cdot 3$$

$$\frac{3\cos(3x)}{\sin(3x)} = 3 \cdot \cot(3x)$$

Ans: E

⑥ Implicit Differentiation

→ treat y as a function of x and don't forget

Chain Rule!!!!!!!

$$\underset{u\ v}{x^5 y} + \underset{u\ v}{x y^2} = \underset{0}{4}$$

Product Rule

$u \cdot v' + u' \cdot v$

$$\underbrace{x^5 \cdot \frac{dy}{dx}} + 5x^4 y + \underbrace{x \cdot 2y \frac{dy}{dx}} + 1y^2 = 0$$

$$x^5 \frac{dy}{dx} + 2xy\frac{dy}{dx} + 5x^4y + y^2 = 0$$
$$-5x^4y - y^2 =$$

$$\frac{\frac{dy}{dx}(\cancel{x^5} + \cancel{2xy})}{(x^5+2xy)} = \frac{-5x^4y - y^2}{(x^5+2xy)}$$

$$\frac{dy}{dx} = \frac{-y(5x^4+y)}{(x^5+2xy)}$$

$\boxed{\text{ANS: C}}$

⑦ Implicit Differentiation

$$x \cdot \sin y + y \cdot \sin x = 0 \qquad x = \frac{\pi}{4}$$
$$u \quad v \qquad\qquad\qquad y = \frac{\pi}{4}$$

$$x \cdot \cos y \cdot \frac{dy}{dx} + 1 \cdot \sin y + y \cdot \cos x + \frac{dy}{dx}\sin x = 0$$

$$\frac{\pi}{4} \cdot \underbrace{\cos\left(\frac{\pi}{4}\right)}\frac{dy}{dx} + \underbrace{\sin\left(\frac{\pi}{4}\right)} + \frac{\pi}{4}\underbrace{\cos\left(\frac{\pi}{4}\right)} + \frac{dy}{dx}\underbrace{\sin\left(\frac{\pi}{4}\right)} = 0$$

$$\underbrace{\left(\frac{\pi}{4}\left(\frac{\sqrt{2}}{2}\right)\right)\frac{dy}{dx}} + \underbrace{\left(\frac{\sqrt{2}}{2}\right) + \frac{\pi}{4}\left(\frac{\sqrt{2}}{2}\right)} + \underbrace{\frac{dy}{dx}\left(\frac{\sqrt{2}}{2}\right)} = 0$$

$$\frac{dy}{dx}\underbrace{\left[\left(\frac{\pi}{4}\right)\frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2}\right]} + \underbrace{\left(\frac{\sqrt{2}}{2} + \left(\frac{\pi}{4}\right)\left(\frac{\sqrt{2}}{2}\right)\right)} = 0$$

$$\frac{dy}{dx}(A) + A = 0$$
$$-A$$

$$\frac{\frac{dy}{dx}(A)}{(A)} = \frac{-A}{(A)} \qquad \frac{dy}{dx} = -1$$

Ans: A

L'Hospital's Rule for Limits:

If:

$$\lim_{x \to a} \frac{f(x)}{g(x)} = \frac{0}{0}, \text{ then}$$

$$\lim_{x \to a} \frac{f(x)}{g(x)} = \lim_{x \to a} \frac{f'(x)}{g'(x)}$$

So, if you get $\frac{0}{0}$, try taking the derivative and substituting again:

Don't do quotient rule, just do f'(x) and g'(x) separately

(11) $$\frac{4\sin\left(\cancel{2}\left(\frac{\pi}{\cancel{2}}\right)-\pi\right)}{\cancel{2}\left(\frac{\pi}{\cancel{2}}\right)-\pi}$$

$$= \frac{4\sin(\pi-\pi)}{\pi-\pi} = \frac{4\sin(0)}{0} = \frac{0}{0}$$

$$\lim_{x\to\frac{\pi}{2}} \frac{\overset{f'(x)}{4\cos(2x-\pi)\cdot\cancel{2}}}{\underset{g'(x)}{\cancel{2}}}$$

$$\frac{4\cos(2x-\pi)}{1}$$

$$\frac{4\cos\left(\cancel{2}\left(\frac{\pi}{\cancel{2}}\right)-\pi\right)}{1}$$

$$\frac{4\cos(0)}{1} = \frac{4(1)}{1} = 4$$

Ans: D

(12) $$\lim_{x \to 0} \frac{\sin(0)-0}{0^3} = \frac{0}{0}$$

$$\lim_{x \to 0} \frac{\cos(x)-1}{3x^2}$$

$$= \frac{\cos(0)-1}{3(0)^2}$$

$$= \frac{1-1}{0} = \frac{0}{0}$$

L'Hospital's Rule Again!

$$\lim_{x \to 0} \frac{-\sin(x)}{6x}$$

$$\frac{-\sin(0)}{6(0)} = \frac{0}{0}$$

L'Hospital's Rule

$$\lim_{x \to 0} \frac{-\cos(x)}{6} = \boxed{\frac{-1}{6}}$$

Ans:

$e^0 = 1$

(13) $\lim\limits_{x \to 0} \dfrac{e^{4x} - e^{2x}}{x} =$

$\dfrac{e^{4(0)} - e^{2(0)}}{0} =$

$\lim\limits_{x \to 0} \dfrac{4e^{4x} - 2e^{2x}}{1} =$

$\dfrac{4e^0 - 2e^0}{1} = \dfrac{4-2}{1} = 2$

Ans: C

(15) $\lim\limits_{x \to \infty} \dfrac{\ln(x)}{x^2} \longrightarrow \dfrac{\ln(\infty)}{(\infty)^2} = \dfrac{\infty}{\infty}$

L'Hospital's Rule Extended:

If $\lim\limits_{x \to \infty} \dfrac{f(x)}{g(x)} = \dfrac{\infty}{\infty}$

you can apply L'Hospital's Rule and see if the following situations apply:

$\dfrac{\#}{\pm\infty} = 0$

$\dfrac{\pm\infty}{\#} = \pm\infty$

$\dfrac{\frac{1}{x}}{2x} = \dfrac{\frac{1}{\infty}}{2(\infty)} = \dfrac{0}{\infty} = 0$

$\boxed{\text{Ans: C}}$

(19) Need to use the limit definition of the derivative:

$$\lim_{h\to 0} \frac{f(x+h)-f(x)}{h} = f'(x)$$

OR

$$\lim_{x\to a} \frac{f(x)-f(a)}{x-a} = f'(x)$$

$$\lim_{h\to 0} \frac{\left[(x+h)^3+3(x+h)+2\right]-\left[(x^3+3x+2)\right]}{h}$$

$$\boxed{f(x)=x^3+3x+2}$$

$$f(x+h)=(x+h)^3+3(x+h)+2$$

$$f'(x)=3x^2+3$$

Ans: C

$$\cot(x)=\frac{\cos(x)}{\sin(x)}$$

$$\csc^2(x)=\frac{1}{\sin^2(x)}$$

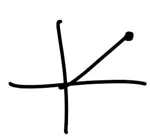
(22)

$$f'(x)=4(-\csc^2(x))+3(\sec^2(x))$$

$$=\frac{-4}{\sin^2(x)}+\frac{3}{\cos^2(x)}$$

$$=\frac{-4}{\left[\sin\left(\frac{\pi}{4}\right)\right]^2}+\frac{3}{\left[\cos\left(\frac{\pi}{4}\right)\right]^2}$$

$\left(\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2}\right)$



$$= \frac{-4}{\left(\frac{\sqrt{2}}{2}\right)^2} + \frac{3}{\left(\frac{\sqrt{2}}{2}\right)^2}$$

$$= \frac{-4}{\left(\frac{2}{4}\right)} + \frac{3}{\left(\frac{2}{4}\right)}$$

$$= \frac{-4(4)}{2} + \frac{3(4)}{2}$$

$$= -8 + 6 = -2$$

Answer = A

(23) $f(x) = \underset{u}{-4x^2}\,\underset{v}{\tan x}$

$$f'(x) = -4x^2(\sec^2(x)) + (-8x)\tan(x)$$

$$= -4\left(\frac{\pi}{4}\right)^2 \frac{1}{\left[\cos\left(\frac{\pi}{4}\right)\right]^2} + (-8)\left(\frac{\pi}{4}\right)\tan\left(\frac{\pi}{4}\right)$$

$$-4\left(\frac{\pi^2}{16}\right) \cdot \frac{1}{\left(\frac{\sqrt{2}}{2}\right)^2} + (-2\pi)(1)$$

$$\frac{-4\pi^2}{16} \cdot \left(\frac{1}{\frac{2}{4}}\right) - 2\pi$$

$$\frac{-4\pi^2}{16} \cdot 2$$

$$\frac{-8\pi^2}{16} - 2\pi = \frac{-\pi^2}{2} - 2\pi\left(\frac{2}{2}\right)$$

$$-\frac{\pi^2}{2} - \frac{4\pi}{2}$$

$$= \frac{-\pi^2 - 4\pi}{2}$$

$$= \frac{\pi(-\pi-4)}{2}$$

$$= \frac{-\pi(\pi+4)}{2}$$

ANS: A

24 Just one to know!

A

Also:

$$\frac{d}{dx}(\sec(x)) = \sec(x)\cdot\tan(x)$$

25 $f(x) = 2\sin x - 3\cos x$

$$f'(x) = 2\cos x - 3(-\sin x)$$

$$= 2\cos x + 3\sin x$$

$$f'(\pi) = 2\cos(\pi) + 3\sin(\pi)$$

$$= 2(-1) + 3(0)$$

$$= -2$$

Ans: D

(26) $f(x) = (x\sqrt{x} - x)(x^2 + x)$

$f(x) = (x^1 \cdot x^{1/2} - x^1)(x^2 + x^1)$

$f(x) = (x^{3/2} - x^1)(x^2 + x^1)$

$= x^{3/2} \cdot x^2 + x^{3/2} \cdot x^1 - x^3 - x^2$

$f(x) = x^{7/2} + x^{5/2} - x^3 - x^2$

$f'(x) = \frac{7}{2}x^{5/2} + \frac{5}{2}x^{3/2} - 3x^2 - 2x$

$f'(1) = \frac{7}{2}(1)^{5/2} + \frac{5}{2}(1)^{3/2} - 3(1)^2 - 2(1)$

$\frac{7}{2} + \frac{5}{2} - 3 - 2$

$6 - 3 - 2 = 1$

Ans: B

(27) $f(x) = \sqrt{\frac{x^3 - 1}{x^3 + 1}}$ $\quad f'(2)$

Chain Rule because it is a composition

Derivative (outside) ·

Derivative (inside)

$\sqrt{u}$ $\qquad u = \frac{x^3 - 1}{x^3 + 1}$ $\begin{matrix} u \\ v \end{matrix}$

outside · inside

$$u^{1/2}$$

$$\frac{1}{2}u^{-1/2}$$

$$\frac{1}{2\sqrt{u}}$$

$$\frac{1}{2\sqrt{\frac{x^3-1}{x^3+1}}}$$

$$\frac{v\cdot u' - u\cdot v'}{v^2}$$

$$\frac{(x^3+1)(3x^2)-(x^3-1)(3x^2)}{(x^3+1)^2}$$

$$\frac{\cancel{3x^5}+3x^2-\cancel{3x^5}+3x^2}{(x^3+1)^2}$$

$$\frac{6x^2}{(x^3+1)^2}$$

$$\boxed{\frac{1}{2\sqrt{\frac{x^3-1}{x^3+1}}}\cdot\frac{6x^2}{(x^3+1)^2}}$$

$$\frac{1}{2\sqrt{\frac{2^3-1}{2^3+1}}}\cdot\frac{6(2)^2}{(2^3+1)^2} =$$

$$\frac{1}{2\sqrt{\frac{7}{9}}}\cdot\frac{24}{81} = \frac{24}{2(81)\left(\frac{\sqrt{7}}{\sqrt{9}}\right)}$$

$$= \frac{12}{81\frac{\sqrt{7}}{\sqrt{9}}}$$

$$= \frac{12}{81\cdot\frac{\sqrt{7}}{3}}$$

$$= \frac{12}{27\sqrt{7}} = \frac{\cancel{3}(4)}{\cancel{3}(9)\sqrt{7}} = \frac{4}{9\sqrt{7}}$$

Ans: C

29) $f(x) = 2e^x - 3^x$

$f'(x) = 2e^x - 3^x \cdot \ln(3)$

$f'(0) = 2e^0 - 3^0 \cdot \ln(3)$

$= 2(1) - (1) \cdot \ln(3)$

$= 2 - \ln(3)$

$\boxed{\text{Ans: C}}$

30) $y = \dfrac{e^x - e^{-x}}{e^x + e^{-x}}$ $\begin{matrix} u \\ v \end{matrix}$

$u = e^x - e^{-x}$

$u' = e^x - e^{-x} \cdot (-1) = e^x + e^{-x}$

$v = e^x + e^{-x}$

$v' = e^x - e^{-x}$

$$\frac{v \cdot u' - u \cdot v'}{v^2}$$

$$= \frac{(e^x+e^{-x})(e^x+e^{-x}) - (e^x-e^{-x})(e^x-e^{-x})}{(e^x+e^{-x})^2}$$

$$= \frac{(e^{2x}+1+1+e^{-2x}) - (e^{2x}-1-1+e^{-2x})}{(e^x+e^{-x})^2}$$

$$\cancel{e^{2x}} + 2 + \cancel{e^{-2x}} - \cancel{e^{2x}} + 2 - \cancel{e^{-2x}}$$

$$\boxed{\frac{4}{(e^x+e^{-x})^2}}$$

$\boxed{\text{Ans: D}}$

31) $S(t) = e^{t^2}$ velocity → 1st derivative

$S'(t) = e^{\boxed{t^2}} \cdot 2t$

$S'(1) = e^{1^2} \cdot 2(1)$

$= e^1 \cdot 2$

$\boxed{= 2e}$ $\boxed{\text{Ans: A}}$

32) Continuous

$\lim_{x \to -1^-} f(x) = \lim_{x \to -1^+} f(x)$

$2(-1) + C \qquad K(-1)^2$

$\boxed{-2 + C = K}$

$-2 + C = -1$
$+2 \qquad\quad +2$

$\boxed{C = 1}$

Differentiable:

$\lim_{x \to -1^-} f'(x) = \lim_{x \to 1^+} f'(x)$

$2 = 2kx$

$2 = 2k(-1)$

$\boxed{2 = -2k}$

$\boxed{k = -1}$

ANS: B

33

$f'(x) = ax^3 + bx^2$

$f'(1) = a(1)^3 + b(1)^2 = -1$

$= \boxed{a + b = -1}$

$f''(x) = 3ax^2 + 2bx$

$f''(1) = 3a(1)^2 + 2b(1) = 6$

$\boxed{3a + 2b = 6}$

$a = \boxed{-1-b}$

$3(-1-b) + 2b = 6$

$-3 - 3b + 2b = 6$

$+3 \qquad +3$

$b = -9$

$$a - 9 = -1$$

$$a = 8$$

$$f'(x) = 8x^3 - 9x^2$$

Anti Derivative

$$f(x) = 2x^4 - 3x^3 + C$$

$$\int_0^1 2x^4 - 3x^3 + C$$

$$\frac{2x^5}{5} - \frac{3x^4}{4} + Cx \Big|_0^1 =$$

$$\frac{2}{5} - \frac{3}{4} + C = \frac{13}{20}$$

$$\frac{8}{20} - \frac{15}{20} + C = \frac{13}{20}$$

$$\frac{-7}{20} + C = \frac{13}{20}$$

$$+\frac{7}{20} \qquad +\frac{7}{20}$$

$$\boxed{C = 1}$$

$$\boxed{f(x) = 2x^4 - 3x^3 + 1}$$

34

a) Approximating a derivative:

Average Rate of change as close as possible

$$\frac{D(8)-D(4)}{8-4} = \frac{26-32}{4}$$

$$= \frac{-6}{4} = -1.5 \ \frac{\text{meters}}{\text{foot}}$$

b)

X	0	2	4	6	8	10
D(X)	10	28	32	30	26	22

$$\frac{1}{10-0}\int_0^{10} D(x)\,dx$$

↳ estimate

Trapezoids

Δx (Avg of the endpoints)

$$(2)(19)+2(30)+2(31)+2(28)+2(24)$$

$$38 + 60 + 62 + 56 + 48$$

$$160 + 104 = 264$$

$$\frac{1}{10}(264) = \boxed{26.4}$$

c) nDeriv$\left(10+15xe^{-.25x}, X, 4\right)$

$= -1.673$

d) $\frac{1}{10}\int_0^{10}(10+15xe^{-.25x})\,dx$

$= \boxed{27.105}$

35

a)

$$\lim_{x\to 2^-} f(x) = \lim_{x\to 2^+} f(x)$$

$$-x^3+3x = 2x^2+kx+p$$

$$-(2)^3+3(2) = 2(2)^2+k(2)+p$$

$$-8+6 = 8+2k+p$$

$$-2 = 8+2k+p$$

$$-8 \quad -8$$

$$-10 = 2k+p$$

$$p = -10-2k$$

$$\lim_{x\to 2^-} f'(x) = \lim_{x\to 2^+} f'(x)$$

$$-3x^2+3 = 4x+k$$

$$-3(2)^2+3 = 4(2)+k$$

$$-3(4)+3 = 8+k$$

$$-9 = 8+k$$

$$-8 \quad -8$$

$$\boxed{k=-17}$$

$$p = -10-2(-17)$$

$$p = -10+34 = \boxed{24}$$

$$f(x) = \begin{cases} -x^3+3x & x \le 2 \\ 2x^2-17x+24 & x > 2 \end{cases}$$

b) where is $f'(x)$ positive?

$f'(x) = -3x^2+3$

$-3x^2+3>0$

$\frac{-3x^2}{-3} > \frac{-3}{-3}$

$x^2 < 1$

$-1 < x < 1$

$f'(x) = 4x-17$

$4x-17>0$

$\frac{4x}{4} > \frac{17}{4}$

$x > 4.25$

Intervals

$(-1,1)$ and $(4.25, \infty)$

$$\int_1^3 f(x)\,dx = \int_1^2 -x^3+3x\,dx + \int_2^3 2x^2-17x+24\,dx$$

$$.75 \quad - 5.833 = \boxed{-5.083}$$