Learning Goal 7.1 Study Guide

The goal: Integrate by using substitution techniques, including both definite and indefinite integrals.

Why do we need substitution? Some anti-derivatives cannot be found by using the power rule or basic trig rules alone.

The process:

Example: Evaluate

$$\int_1^2 (12x+8)e^{3x^2+4x}\,dx$$

1) Choose an expression to substitute (call it u)

2) If it is a definite integral, find new bounds by substituting.

Choose $u = 3x^2+4x$

$u = 3(1)^2 + 4(1) = 7$

$u = 3(2)^2 + 4(2) = 20$

3) Find $\frac{du}{dx}$, then solve for dx and substitute

$$\frac{du}{dx} = 6x+4$$

$$dx = \frac{du}{(6x+4)}$$

4) Simplify! If you did it correctly, there will be no more x's!

$$\int_7^{20} (12x+8)e^u \frac{du}{(6x+4)} = \int_7^{20} 2\cancel{(6x+4)}e^u \frac{du}{\cancel{(6x+4)}}$$

5) Find the anti-derivative with respect to u and evaluate using FTC

(If there are no bounds, sub in expression for u and add +C)

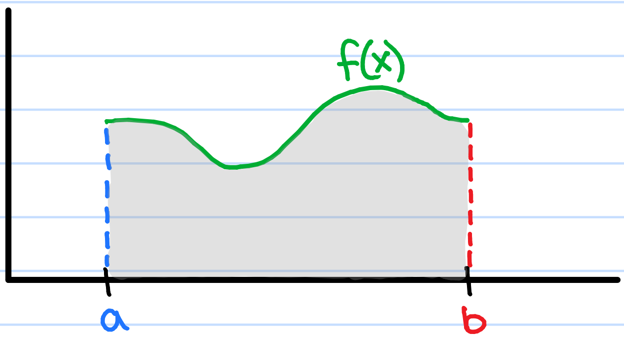
$$\int_7^{20} 2e^u\,du =$$

$$2e^u \Big|_7^{20} = 2e^{20} - 2e^7$$

Learning Goal 7.2 Study Guide

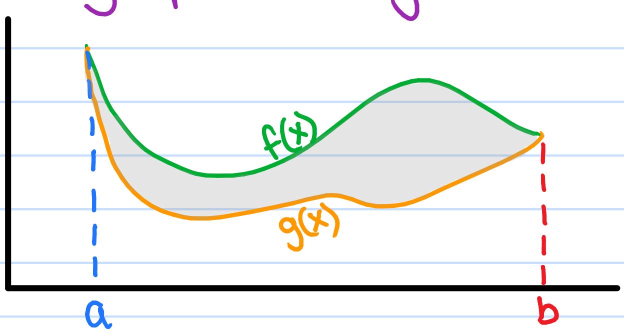
The goal: Use integrals to find area of a region in the xy-plane.

Setting up an integral with x-axis as a boundary:



$$\int_a^b f(x)\,dx$$

Setting up an integral to represent area between curves:

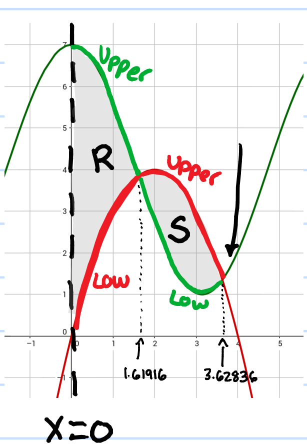


$$\int_a^b \underset{\text{Upper}}{f(x)} - \underset{\text{Lower}}{g(x)}\,dx$$

Important questions to ask:

→ Which is the "upper" function and "lower" function?
→ If a and b are not stated, where do the functions meet?
→ Do I need multiple integrals?

Example: Write an integral or integrals to represent the area enclosed by $x=0$, $y=3\cos x$ and $y=-(x-2)^2+4$



The line $x=0$ is a vertical line. The graphs meet twice at $x=1.61916$ and $x=3.62836$ (found with a calculator)

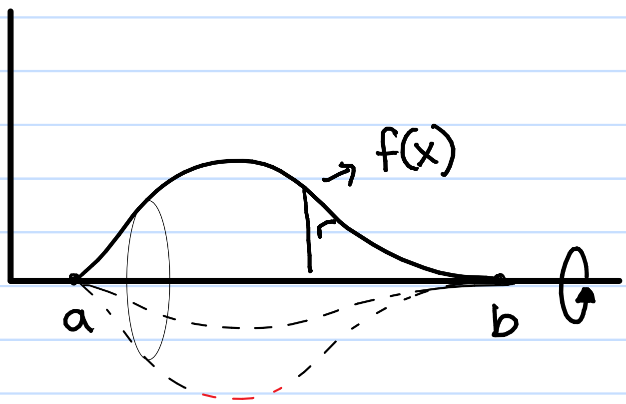
Therefore, the area can be expressed with 2 integrals:

$$\underbrace{\int_0^{1.61916} \underset{\text{Upper}}{(3\cos x)} - \underset{\text{Lower}}{(-(x-2)^2+4)}\,dx}_{R} + \underbrace{\int_{1.61916}^{3.62836} \underset{\text{Upper}}{(-(x-2)^2+4)} - \underset{\text{Lower}}{(3\cos x)}\,dx}_{S}$$

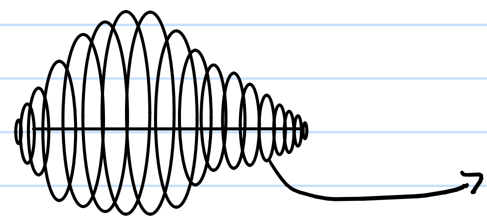
Learning Goal 7.3 Study Guide

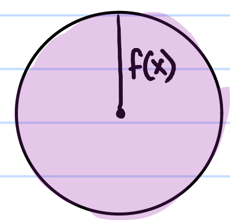
The goal: Setup and evaluate integrals representing the volume of 3D solids

Revolving Disks Around the x-axis



Think about a bunch of circles stacked on top of each other.



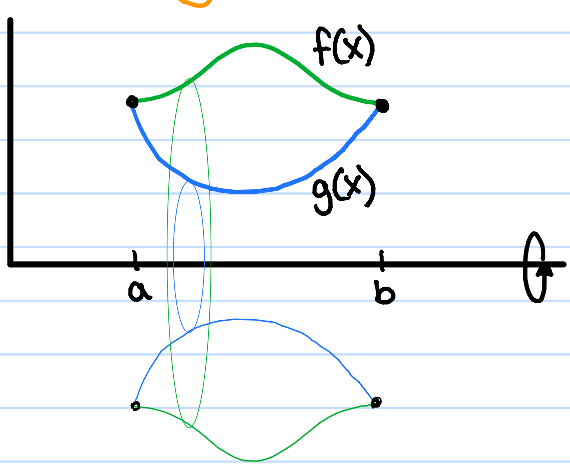


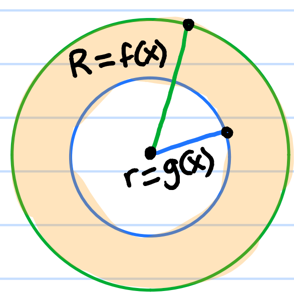
Each circle has a radius of f(x), so we need to think about adding up lots of circle areas:

$\pi r_1^2 + \pi r_2^2 + \pi r_3^2 + \pi r_4^2$ .... which we write as an integral:

$\boxed{\int_a^b \pi (f(x))^2 \, dx}$ or $\pi \int_a^b f(x)^2 \, dx$

Revolving Washers Around the x-axis:





The area of the orange washer is $\pi R^2 - \pi r^2 = \pi (f(x))^2 - \pi (g(x))^2$

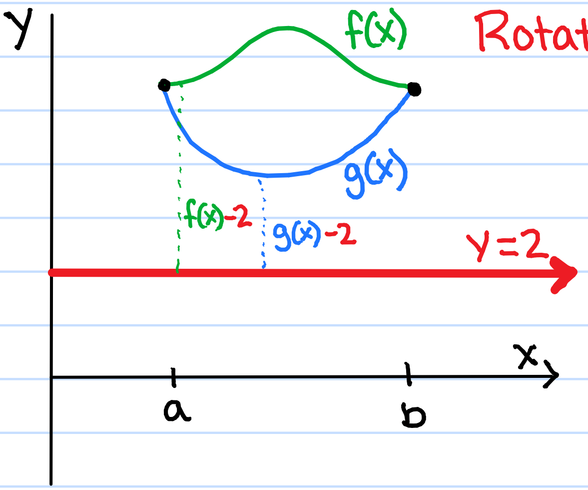
Each washer has area of $\pi (f(x))^2 - \pi (g(x))^2$ so the total area can be expressed as:

$$\pi \int_a^b f(x)^2 - g(x)^2 \, dx$$

where f(x) is the outer radius & g(x) is the inner radius

Rotating Around Other Horizontal Lines

Rotating around line below region: Example $y=2$

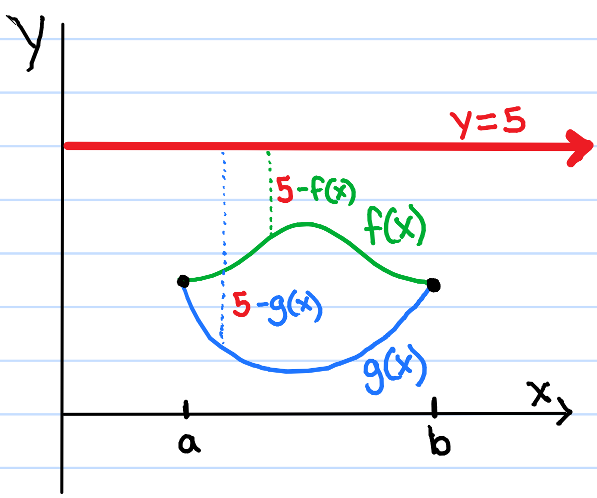


Outer Radius $= f(x)-2$

Inner Radius $= g(x)-2$

$$\text{Volume} = \int_a^b \pi\left(f(x)-2\right)^2 - \pi\left(g(x)-2\right)^2\,dx$$

Rotating around a line above the region: Example: $y=5$



Note: The outer and inner functions switch!

Outer Radius: $5-g(x)$

Inner Radius: $5-f(x)$

$$\text{Volume} = \int_a^b \pi\left(5-g(x)\right)^2 - \pi\left(5-f(x)\right)^2\,dx$$