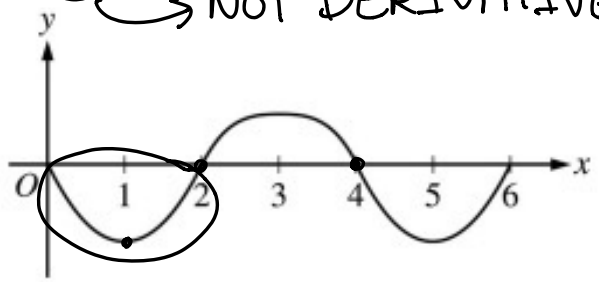
A Quick Guide to Curve Sketching:

graph feature		f(x)	f'(x)	f''(x)	Notes
rising (L to R)		slope > 0	+		
falling (L to R)		slope < 0	−		
extrema	maximum	slope = 0	= 0 + on L, − on R	− at x_{max}	derivative may not exist at a max or min, e.g. ∧ or ∨
extrema	minimum	slope = 0	= 0 − on L, + on R	+ at x_{min}	
inflection pt.		Curvature changes: ∪ ↔ ∩		= 0 potential inflection point	
concave up		∪	−	+	+	
concave down		∩	+	−	−	

Note: You can use this chart to help you solve LOTS of problems, even if you are not asked to find the curve.

Example 1: The graph of f(x) is shown below.

NOT DERIVATIVE! Original



2,4 POIs

1,3,5 critical numbers

Make a sign chart for this function. What can you say about the first and second derivative.

X		1		2		3		4		5		
f(x)												→ Points
f'(x)	−	0	+		+	0	−		−	0	+	→ slope
f''(x)	+		+	0			−	0			+	+

Example 2:

The first derivative of a function is given by the equation below:

$$f'(x) = x(x+2)(x-3)^2$$

✳ This is telling me the slope!!!

a) Find the values where the function has a minimum or maximum:

x	-2	-1	0	1	3
$f'(x)$	$+$	0	$-$	0	$+$	0	$+$

min at $x = 0$
max at $x = -2$
not max or min

b) On what interval(s) is the function increasing? On what interval(s) is the function decreasing?

I: $(-\infty, -2) \cup (0, 3) \cup (3, \infty)$ Not including 3

D: $(-2, 0)$

Example 3:

Find an interval where the function below is both decreasing and concave up:

$f(x) = x^3 + 7x^2 + 8x$

$f'(x) = 3x^2 + 14x + 8$ → $0 = 3x^2 + 14x + 8$

Critical Points:

$= (3x^2 + 12x) + (2x + 8)$

$3x(x+4) + 2(x+4)$

$(3x+2)(x+4)$

$x = -\frac{2}{3}$ $x = -4$

$f''(x) = 6x + 14$

x		-4		$\frac{-14}{6}$		$-\frac{2}{3}$	
$f(x)$							
$f'(x)$	$+$		0	$-$		0	$+$
$f''(x)$		$-$		0	$+$		

POIs

$0 = 6x + 14$

$\frac{-6x}{-6} = \frac{14}{-6}$

Interval where decreasing and Concave up: $\left(-\frac{14}{6}, -\frac{2}{3}\right)$