# UNIT 1: LIMITS AND CONTINUITY REVIEW PACKET KEY

Name ______________________________________ Date _______________________ Score ___________

**Find each limit. Show your work.**

1. 6x + 4
2. $-\infty$ (Take a look at the graph on desmos – I included pi/2 in red to see where to examine)
https://www.desmos.com/calculator/xbwog0ol5l

3. $\lim_{x\to\infty} \frac{5x^3+27}{20x^2+10x+9}$

$= \lim_{x\to\infty} \frac{x}{4}$

$= \infty$

4. $\frac{\sqrt{10}}{10}$

5. $\frac{-2}{x^2}$

6. $\lim_{x\to-\infty} \frac{1}{x^2} = 0$

7. $\lim_{x\to 3} \frac{x-3}{x^2-2x-3}$

$= \lim_{x\to 3} \frac{(x-3)}{(x-3)(x+1)}$

$= \frac{1}{4}$

8. $\lim_{x\to\infty} \frac{2x^2+1}{(2-x)(2+x)}$

$= \lim_{x\to\infty} \frac{2x^2+1}{4-x^2}$

$= -2$

9. 1 (take a look on desmos here to confirm: https://www.desmos.com/calculator/6dhzno9bra)

*Note*: *Algebraically, you can first rewrite this in terms of sin and cos, then divide numerator and denominator by x. Then take the limit of numerator and denominator. Voila!*

10. –219

**Use the piecewise function to find the indicated limits or state that the limit does not exist.**

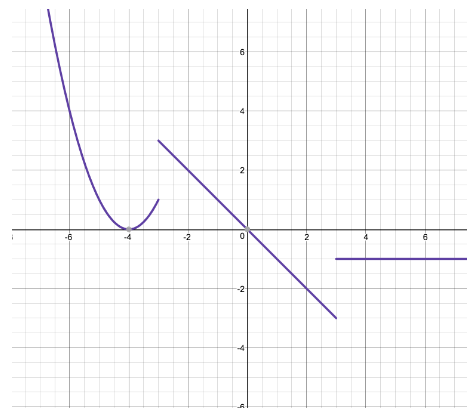
9. $f(x) = \begin{cases} x^2 + 1 & \text{when } x < 2 \\ 3x + 1 & \text{when } x \geq 2 \end{cases}$

a. $\lim_{x \to 2^-} f(x) = 5$

b. $\lim_{x \to 2^+} f(x) = 7$

c. $\lim_{x \to 2} f(x) =$ Does not exist

**Use the function *f*(*x*) = | *x* – 3 | for questions 10-15.**



10. Graph *f*(*x*) in the space provided, then write *f*(*x*) as a piecewise function.

$f(x) = \begin{cases} (x+4)^2, & x \leq -3 \\ -x, & -3 < x < 3 \\ -1, & x \geq 3 \end{cases}$

11. $\lim_{x \to 3^-} f(x) =$ -3

12. $\lim_{x \to 3^+} f(x) =$ -1

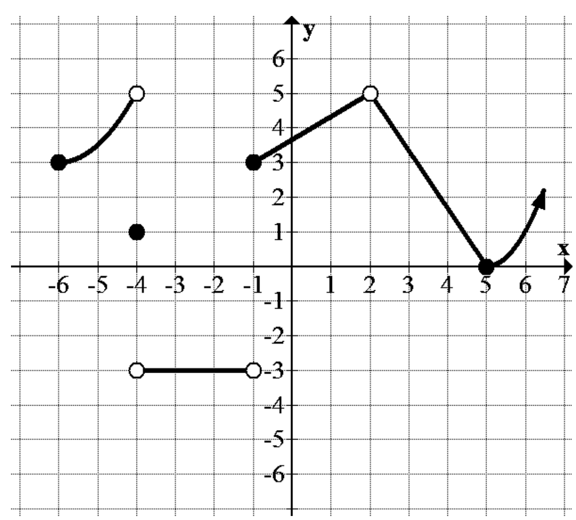
13. $\lim_{x \to 3} f(x) =$ does not exist

14. f(3) = -1

15. Is *f*(*x*) continuous at *x* = 3? __no__ Justify your answer. $\lim_{x \to 3^-} f(x) \neq \lim_{x \to 3^+} f(x) \neq f(3)$

The limit at x = 3 is not equal to f(3).

**Find each limit or function value.**




16. $\lim_{x \to -1^-} f(x) =$ __–3__

17. $\lim_{x \to -1^+} f(x) =$ __3__

18. $\lim_{x \to -4^-} f(x) =$ __5__

19. $\lim_{x \to -4^+} f(x) =$ __–3__

20. $\lim_{x \to -4} f(x) =$ Does not exist

21. *f*(-4) = __1__

22. $\lim_{x \to 2^-} f(x) =$ __5__

23. $\lim_{x \to 2^+} f(x) =$ __5__

**Identify any horizontal and/or vertical asymptotes. Show your work using proper notation to justify your answers.**

24. $f(x) = \dfrac{4}{x^2 - 1}$

Horizontal Asymptote(s):

$$\lim_{x\to\infty} \frac{4}{x^2 - 1} = 0$$

$$\lim_{x\to-\infty} \frac{4}{x^2 - 1} = 0$$

One Horizontal Asymptote: $y = 0$

Vertical Asymptote(s):

$x^2 - 1 = 0$

$x = \pm 1$

Two Vertical Asymptotes: $x = 1$ and $x = -1$.

**Identify all points of discontinuity and name the type of discontinuity.**

25. $f(x) = \dfrac{x^2 + x}{x}$

$f(x) = \dfrac{x(x+1)}{x} = (x+1)$.

$f(x)$ is discontinuous at $x = 0$. Removable discontinuity.

26. $f(x) = \begin{cases} \sin x & x < 0 \\ 1 - x & 0 \le x \le 1 \\ \ln x & x > 1 \end{cases}$

$\lim_{x\to 0^-} \sin x = 0$ $\quad$ $\lim_{x\to 0^+} (1 - x) = 1$ $\quad$ $f(x)$ is discontinuous at $x = 0$. Jump discontinuity.

$\lim_{x\to 1^-} (1 - x) = 0$ $\quad$ $\lim_{x\to 1^+} (\ln x) = 0$ $\quad$ $f(x)$ is continuous at $x = 1$.

**Use the Intermediate Value Theorem to determine if the given function has a zero in the specified interval. Do not attempt to locate any zeros.**

27. $f(x) = x^2 + 1$ on $[-2, 4]$

Since $f$ is a polynomial, it is continuous on $[-2, 4]$. Because $f(-2) = 5$ and $f(4) = 17$ have the same sign, the Intermediate Value Theorem indicates that the function will not have a zero in the specified interval.

**Solve.**

28. The graph of $y = \dfrac{2x^2 + 2x + 3}{4x^2 - 4x}$ has

(A) a horizontal asymptote at $y = ½$, but no vertical asymptote.

**(B) a horizontal asymptote at $y = ½$ and two vertical asymptotes, at $x = 0$ and $x = 1$.**

(C) a horizontal asymptote at $x = 2$, but no vertical asymptote.

(D) a horizontal asymptote at $y = ½$ and two vertical asymptotes, at $x = \pm 1$.

29. Let $f(x) = \begin{cases} \dfrac{x^2 + x}{x} & \text{if } x \neq 0 \\ 1 & \text{if } x = 0 \end{cases}$

Which of the following statements are true?

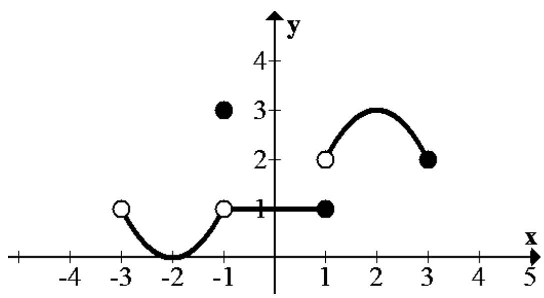
I. $f(0)$ exists.

II. $\lim_{x \to 0} f(x)$ exists.

III. $f$ is continuous at $x = 0$.

(A) I only (B) II only (C) I and II **(D) I, II, III**

30.




The figure above shows the graph of a function $f$ with domain $-3 < x \leq 3$. Which of the following statements are true?

I. $\lim_{x \to 1^-} f(x)$ exists.

II. $\lim_{x \to 1^+} f(x)$ exists.

III. $\lim_{x \to 1} f(x)$ exists.

(A) I only (B) II only **(C) I and II** (D) I, II, III

**More Continuity Problems**

In Problems 1–2, determine whether the function *f* is continuous at the given values of *c*. Justify your answer.

1. $f(x) = \begin{cases} -2x+1 & x < -1 \\ 2 & x = -1 \\ 2x+1 & -1 < x < 1 \\ x^2+2 & x \ge 1 \end{cases}$

$c = -1,\ c = 1$

$f(x)$ is not continuous at $c = -1$ because $\lim_{x\to -1^-} f(x) \ne \lim_{x\to -1^+} f(x)$. They equal 3 and 2 respectively.

$f(x)$ is continuous at $c = 1$ because $\lim_{x\to 1} f(x) = f(1) = 3$

2. $f(x) = \begin{cases} 1 & x \le -1 \\ -x & -1 < x < 0 \\ 1 & x = 0 \\ -x & 0 < x < 1 \\ 1 & x \ge 1 \end{cases}$

$c = -1,\ c = 0,\ c = 1$

$f(x)$ is continuous at $c = -1$ because $\lim_{x\to -1} f(x) = f(-1) = 1$

$f(x)$ is not continuous at $c = 0$ because $\lim_{x\to 0} f(x) \ne f(0)$.

They equal 0 and 1 respectively.

$f(x)$ is not continuous at $c = 1$ because

$\lim_{x\to 1^-} f(x) \ne \lim_{x\to 1^+} f(x)$. They equal –1 and 1, respectively.

Solve.

3. For all but two choices of *a*, the function $y = \begin{cases} x^3 & \text{if } x \le a \\ x^2 & \text{if } x > a \end{cases}$ will be discontinuous at the point $x = a$. For what values of *a* will $f(x)$ be continuous at *a*?

$a^3 = a^2$

$a^3 - a^2 = 0$

$a^2(a-1) = 0$

$a = 0, a = 1$

4. At what value or values of *x* is the function $y = \begin{cases} x+2 & \text{if } x \le -1 \\ x^2 & \text{if } -1 < x < 1 \\ 3-x & \text{if } x \ge 1 \end{cases}$ discontinuous?

Discontinuous at $x = 1$.

5. Find the value of the constants, *c*, that make the function $y = \begin{cases} c^2 - x^2 & \text{if } x < 2 \\ 2(c-x) & \text{if } x \ge 2 \end{cases}$ continuous on $(-\infty, \infty)$.

$c^2 - 2^2 = 2(c-2)$

$c^2 - 4 = 2c - 4$

$c^2 - 2c = 0$

$c(c-2) = 0$

$c = 0, c = 2$