Math 101: Sequences and Series
Practice Problem Set – Answer Key

1. Which of the following is an arithmetic sequence?
   a) 4, 8, 16…
   b) 100, 20, 4…
   c) 2, 4, 16…
   d) 5, 7, 9…

   Answer: d. Arithmetic sequences increase or decrease by continually adding the same value to create the sequences. The first sequence is being multiplied by two each time, the second is being divide by 5 each time, the third is being squared each time, and the final is increasing by 2 each time.

2. Which of the following is an arithmetic sequence?
   a) 10, 20, 30…
   b) 3, 9, 27…
   c) $\frac{1}{2}, \frac{1}{4}, \frac{1}{8} \ldots$
   d) 5, -10, 20…

   Answer: a. Arithmetic sequences increase or decrease by continually adding the same value to create the sequences. The first sequence is increasing by 10 each time, the second is being multiplied by 3 each time, the third is being divided by 2 each time, and the final is being multiplied by -2 each time.

3. Which of the following is an arithmetic sequence?
   a) 0.2, 0.4, 0.8…
   b) 5, 4, 3…
   c) $-4, 8, -16 \ldots$
   d) 10, 5, 2…

   Answer: b. Arithmetic sequences increase or decrease by continually adding the same value to create the sequence. The first sequence is being multiplied by 2 each time, the second is having -1 added each time, the third is being multiplied by -2 each time, and the final does not have a discernable pattern.

4. What is the common difference in the sequence and 29th term? 5, 7, 9…

   Answer: common difference = 2

$$a_n = dn + a_0 \quad or \quad a_n = d(n-1) + a_1$$
$$a_n = d(n-1) + a_1$$
$$a_{29} = 2(29-1) + 5$$
$$a_{29} = 2(28) + 5 = 56 + 5 = 61$$

5. What is the common difference of the sequence and the 15th term? 8, 14, 20…

   Answer: common difference= 6

   $$a_n = dn + a_0 \quad or \quad a_n = d(n-1) + a_1$$
   $$a_n = d(n-1) + a_1$$
   $$a_{15} = 6(15-1) + 8$$
   $$a_{15} = 6(14) + 8 = 84 + 8 = 92$$

6. What is the 46th term in the sequence, 99, 98.5, 98… ?

   Answer: common difference = -0.5

   $$a_n = dn + a_0 \quad or \quad a_n = d(n-1) + a_1$$
   $$a_n = d(n-1) + a_1$$
   $$a_{46} = -0.5(46-1) + 99$$
   $$a_{46} = -0.5(45) + 99 = -22.5 + 99 = 76.5$$

7. Which of the following is a geometric sequence?
   a) 4, 8, 16…
   b) 100, 80, 60…
   c) 3, 6, 18…
   d) 4, 14, 24…

   Answer: a. Geometric sequences increase or decrease by continually multiplying by the same value to create the sequences. The first sequence is being multiplied by two each time, the second is being reduced by subtracting 20, the third is being multiplied by different values, and the final is increasing by 10 each time.

8. Which of the following is a geometric sequence?
   a) 5, 10, 15…
   b) 100, 50, 25…
   c) 3, 7, 11…
   d) 4, 2, 8…

   Answer: b. Geometric sequences increase or decrease by continually multiplying by the same value to create the sequences. The first sequence is adding 5 each time, the second is being divided by 2 which is equivalent to multiplying by ½ , the third is being increased by 4 each time, and the final does not have a discernable pattern.

9. When does a sequence become a series?

Answer: A sequence becomes a series when the terms of the sequence are added together to obtain a sum.

10. Which of the following is a geometric series?
    a) 5 + 10 + 15 + 20
    b) 4, 8, 16…
    c) 8 + 24 + 72 + 216
    d) 9, 12, 15…

Answer: c. Geometric series are the sum of a set of numbers that increase or decrease by continually multiplying by the same value to create the set of numbers. The first set is a sum but it is adding 5 each time, the second is not a series because although the values are being multiplied by 2 each time they are not added together, the third is being multiplied by 3 each time and is then added together, and the final is only being increased by three each time.

11. Which of the following is a geometric series?
    a) 2 + 3 + 4 + 5
    b) 40 × 20 × 10 × 5
    c) 2 + 4 + 16 + 256
    d) 3 + 6 + 12 + 24

Answer: d. Geometric series are the sum of a set of numbers that increase or decrease by continually multiplying by the same value to create the set of numbers. The first set is a sum but it is adding 1 each time, the second is not a series because although the values are being multiplied by ½ each time they are not added together, the third is being increased exponentially each time and is then added together, and the final is being multiplied by two each time and then summed.

12. What is the sum of the first seven terms of the sequence 1, 3, 9, 27…?

Answer:

$$\sum_{n=1}^{n} a_1 \times r^{n-1} = \frac{a_1(1-r^n)}{1-r}$$

$$\frac{a_1(1-r^n)}{1-r} = \frac{1(1-3^7)}{1-3} = \frac{1(1-2187)}{1-3}$$

$$\frac{a_1(1-r^n)}{1-r} = \frac{-2186}{-2} = 1093$$

13. What is the sum of the first ten terms of the sequence 4, 12, 36, 108…?

Answer:

$$\sum_{n=1}^{n} a_1 \times r^{n-1} = \frac{a_1(1-r^n)}{1-r}$$

$$\frac{a_1(1-r^n)}{1-r} = \frac{4(1-3^{10})}{1-3} = \frac{4(1-59049)}{1-3}$$

$$\frac{a_1(1-r^n)}{1-r} = \frac{-236192}{-2} = 118096$$

14. What is the sum of the first six terms of the sequence 1000, 500, 250...?

Answer:

$$\sum_{n=1}^{n} a_1 \times r^{n-1} = \frac{a_1(1-r^n)}{1-r}$$

$$\frac{a_1(1-r^n)}{1-r} = \frac{1000\left(1-\left(\frac{1}{2}\right)^6\right)}{1-\left(\frac{1}{2}\right)} = \frac{1000\left(1-\frac{1}{64}\right)}{1-\frac{1}{2}}$$

$$\frac{a_1(1-r^n)}{1-r} = \frac{1000\left(\frac{63}{64}\right)}{\frac{1}{2}} = 1968.75$$

15. At which term does the following series begin?

$$\sum_{m=2}^{5} 2m$$

Answer: $m = 2$ indicates that the series begins at the second term

16. At which term does the following series begin?

$$\sum_{m=1}^{10} 4m$$

Answer: $m = 1$ indicates that the series begins at the first term

17. At which term does the following series begin?

$$\sum_{m=5}^{7} 3m$$

Answer: $m = 5$ indicates that the series begins at the fifth term

18. What is the rule for the following series?

$$\sum_{m=0}^{7} m!$$

Answer: The rule is $m!$

19. What is the rule for the following series?

$$\sum_{m=0}^{9} 4m + 5$$

Answer: The rule is $4m + 5$

20. Write the series that is represented by the summation notation given, identify the rule, and then solve?

$$\sum_{f=1}^{3} 5^f$$

Answer: $5 + 25 + 125 = 155$, rule $= 5^f$ This summation notation indicates a series of the first three terms of a sequence starting at 5^1 and being raised to a higher power of 5 each time, ending at 5^3.

21. Write the series that is represented by the summation notation given, identify the rule, and then solve?

$$\sum_{p=1}^{6} 3^p$$

Answer: $3 + 9 + 27 + 81 + 243 + 729 = 1029$, rule= 3^p. This summation notation indicates a series of the first six terms of a sequence starting at 3^1 and being raised to a higher power each time, ending at 3^6.

22. Write the series that is represented by the summation notation given, identify the rule and then solve?

$$\sum_{m=2}^{5} 2m$$

Answer: $2(2) + 2(3) + 2(4) + 2(5) = 28$, rule= $2m$. This summation notation indicates a series of the four terms where 2 is being multiplied by 2, 3, 4, and 5.

23. What is the summation notation for 10 + 20 + 40 + 80 + 160 + 320?

Answer:

$$\sum_{n=1}^{n} a_1 \times r^{n-1}$$
$$\sum_{n=1}^{6} 10 \times 2^{n-1}$$

This series starts at 10 and is the sum of the first 6 terms. The rule is that each number is multiplied by 2 each time.

24. What is the summation notation for 8 + 32 + 128 + 512?

Answer:

$$\sum_{n=1}^{n} a_1 \times r^{n-1}$$
$$\sum_{n=1}^{4} 8 \times 4^{n-1}$$

This series starts at 8 and is the sum of the first 4 terms. The rule is that each number is multiplied by 4 each time.

25. What is the summation notation for 3 + 15 + 45?

Answer:

$$\sum_{n=1}^{n} a_1 \times r^{n-1}$$
$$\sum_{n=1}^{3} 3 \times 5^{n-1}$$

This series starts at 3 and is the sum of the first 3 terms. The rule is that each number is multiplied by 5 each time.

26. Evaluate:

$$\sum_{n=1}^{4} 3n - 2$$

Answer: $(3n_1 - 2) + (3n_2 - 2) + (3n_3 - 2) + (3n_4 - 2)$
$(3(1) - 2) + (3(2) - 2) + (3(3) - 2) + (3(4) - 2)$

$$(3-2)+(6-2)+(9-2)+(12-2)=1+4+7+10=22$$

27. Evaluate:

$$\sum_{n=1}^{6} p+3$$

Answer: $(p_1+3)+(p_2+3)+(p_3+3)+(p_4+3)+(p_5+3)+(p_6+3)$

$(1+3)+(2+3)+(3+3)+(4+3)+(5+3)+(6+3)$

$4+5+6+7+8+9=39$

28. What is the sum of all numbers from 1 to 300?

Answer:

$$\sum_{n=1}^{n} a_n = \left(\frac{n}{2}\right)(a_1+a_n)$$

$$\left(\frac{n}{2}\right)(a_1+a_n) = \left(\frac{300}{2}\right)(1+300) = 150(301) = 45{,}150$$

29. What is the sum of all numbers from 1 to 99?

Answer:

$$\sum_{n=1}^{n} a_n = \left(\frac{n}{2}\right)(a_1+a_n)$$

$$\left(\frac{n}{2}\right)(a_1+a_n) = \left(\frac{99}{2}\right)(1+99) = 49.5(100) = 4{,}950$$

30. What is the sum of the first 10 terms of the arithmetic series that begins at 2 and has a common difference of 3?

Answer:

$$\sum_{n=1}^{n} a_n = \left(\frac{n}{2}\right)(a_1+a_n)$$

$$\sum_{n=1}^{10} n+3 = \left(\frac{10}{2}\right)(2+(10+3))$$

$$\sum_{n=1}^{10} n+3 = (5)(15) = 75$$

31. What is the sum of the first 20 terms of the arithmetic series that begins at 7 and has a common difference of 8?

Answer:

$$\sum_{n=1}^{n} a_n = \left(\frac{n}{2}\right)(a_1 + a_n)$$

$$\sum_{n=1}^{20} n + 8 = \left(\frac{20}{2}\right)(7 + (20 + 8))$$

$$\sum_{n=1}^{20} n + 8 = (10)(35) = 350$$

32. What is the sum of the first 17 terms of the arithmetic series that begins at 5 and has a common difference of -4?

Answer:

$$\sum_{n=1}^{n} a_n = \left(\frac{n}{2}\right)(a_1 + a_n)$$

$$\sum_{n=1}^{17} n - 4 = \left(\frac{17}{2}\right)(5 + (17 - 4))$$

$$\sum_{n=1}^{17} n - 4 = (8.5)(18) = 153$$

33. What does the sum of this infinite geometric series approach? 100, 50, 25…

Answer:

$$\sum_{n=1}^{\infty} a_1 \times r^{n-1} = \frac{a_1}{1-r}$$

$$\frac{a_1}{1-r} = \frac{100}{1-\frac{1}{2}} = \frac{100}{0.5} = 200$$

34. What does the sum of this infinite geometric series approach? 99, 33, 11…

Answer:

$$\sum_{n=1}^{\infty} a_1 \times r^{n-1} = \frac{a_1}{1-r}$$

$$\frac{a_1}{1-r} = \frac{99}{1-\frac{1}{3}} = \frac{99}{\frac{2}{3}} = 148.5$$

35. What does the sum of this infinite geometric series approach? 500, 100, 20, 4…

Answer:

$$\sum_{n=1}^{\infty} a_1 \times r^{n-1} = \frac{a_1}{1-r}$$

$$\frac{a_1}{1-r} = \frac{500}{1-\frac{1}{5}} = \frac{500}{\frac{4}{5}} = 625$$