AP Calculus AB HW: Week 3: 9/10 - 9/14

Textbook - 1.1 (#4, 5, 10, 11, 18, 22, 27, 31, 32, 38, 41, 42, and ALL AP Practice Problems)

Textbook - 1.2 (#7, 10, 36, 37, 40, 41, 47, 48, 54, 55, 56, 61, and 65)

Please find answers to ODD problems in the back of your textbook.

1.1) Limits of Functions using Numerical and Graphical Techniques

4. $\boxed{\text{False}}$. The limit L of a function $y = f(x)$ as x approaches the number c **does not** depend on the value of f at c.

10. The values in the table below suggest that the value of $f(x) = x^2 - 2$ can be made "as close as we please" to -1 by choosing x "sufficiently close" to -1. It therefore appears that

$$\lim_{x \to -1} (x^2 - 2) = \boxed{-1}.$$

x	-1.1	-1.01	-1.001	$\to -1 \leftarrow$	-0.999	-0.99	-0.9
$f(x) = x^2 - 2$	-0.79	-0.9799	-0.997999	$f(x)$ approaches -1	-1.001999	-1.0199	-1.19

18. The graph suggests that the value of f approaches 4 as x approaches 2 from the left and as x approaches 2 from the right. Thus,

(a) $\lim_{x \to 2^-} f(x) = \boxed{4}$;

(b) $\lim_{x \to 2^+} f(x) = \boxed{4}$; and

(c) $\lim_{x \to 2} f(x) = \boxed{4}$.

22. The graph suggests that, as x approaches c from the left,

$$\lim_{x \to c^-} f(x) = 1,$$

while, as x approaches c from the right,

$$\lim_{x \to c^+} f(x) = 1.$$

Because the two one-sided limits are equal, it follows that

$$\lim_{x \to c} f(x) = \boxed{1}.$$

32. The graph of f shown below suggests that, as x approaches 2 from the left,

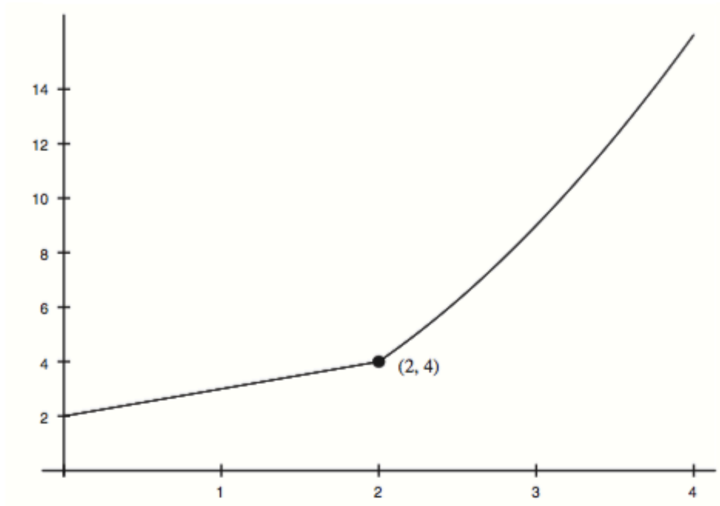
$$\lim_{x \to 2^-} f(x) = 4,$$

while, as x approaches 2 from the right,

$$\lim_{x \to 2^+} f(x) = 4.$$

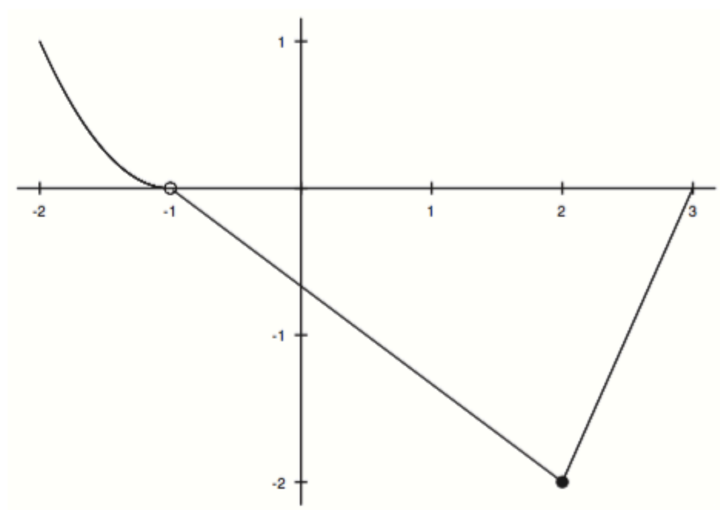
Because the two one-sided limits are equal, it follows that

$$\lim_{x \to 2} f(x) = \boxed{4}.$$



38. Answers will vary. Below is the graph of a function f for which

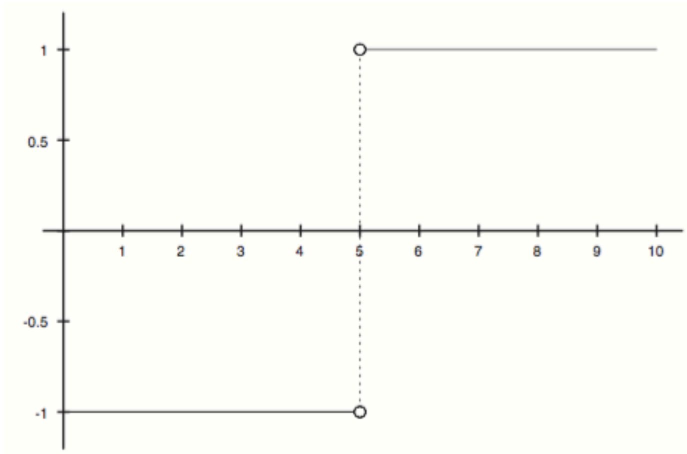
$$\lim_{x \to -1} f(x) = 0; \quad \lim_{x \to 2^-} f(x) = -2; \quad \lim_{x \to 2^+} f(x) = -2; \quad f(-1) \text{ is not defined}; \quad f(2) = -2.$$



42. The table of values below suggests $\lim_{x \to 5^-} \frac{|x-5|}{x-5} = \boxed{-1}$.

x	4.9	4.99	4.999	$\to 5$
$f(x) = \frac{\|x-5\|}{x-5}$	-1	-1	-1	$f(x)$ approaches -1

Alternately, the graph below suggests $\lim_{x \to 5^-} \frac{|x-5|}{x-5} = \boxed{-1}$.



AP Practice Problems

1. The notation $x \to 1^-$ used in the one-sided limit, $\lim_{x \to 1^-} \sqrt{1-x}$, indicates that x is approaching 1 from the left. Using a table, values slightly less than 1 (but getting closer to 1) should be chosen. The answer is B.

2. The notation $x \to -2^+$ used in the one-sided limit, $\lim_{x \to -2^+} x^3 = -8$, indicates that x is approaching –2 from the right. Values slightly greater than (to the right of) –2 are getting closer to –8. This is a right-sided limit. The answer is D.

3. From *The Idea of a Limit* in this section, $\lim_{x \to c} f(x) = L$ is read "the limit as x approaches c of $f(x)$ is equal to the number L." Using this idea, "the limit as x approaches 0 of $f(x) = \cos(x)$ is equal to the number 1" is written symbolically as $\lim_{x \to 0} \cos(x) = 1$. The answer is C.

4. In the graph of f that is provided, $\lim_{x \to 3^-} f(x) = 1$. For values slightly less than 3, as x gets closer to 3 (without being equal to 3), the values of $f(x)$ are getting closer to 1. Similarly, $\lim_{x \to 3^+} f(x) = 1$. For values slightly more than 3, as x gets closer to 3 (without being equal to 3), the values of $f(x)$ are getting closer to 1. The limit L of a function $y = f(x)$ as x approaches a number c exists if and only if both one-sided limits exist at c and both one-sided limits are equal. Since $\lim_{x \to 3^-} f(x) = \lim_{x \to 3^+} f(x) = 1$, we conclude that $\lim_{x \to 3} f(x) = 1$. The answer is A.

5. Selection A is not true since $\lim_{x \to -1^-} f(x) = \lim_{x \to -1^+} f(x)$. Selection C is not true since $\lim_{x \to 1^-} f(x) \neq \lim_{x \to 1^+} f(x)$ and, consequently $\lim_{x \to 1} f(x)$ does not exist. Selection D is not true since the value of $y = f(x)$ is not defined for $x = 4$. Selection B is true since the left-sided $\lim_{x \to 1^-} f(x)$ is the same number as the value of $y = f(x)$ for $x = 1$. The answer is B.

6. From the information provided about function f, we observe that as x approaches 1 from the left, the value of f is 0, and as x approaches 1 from the right, the value of f is 0.9. The table also indicates that $f(1) = 0$, but this value has no impact on the limit as x approaches 1. Since the table suggests that the left-sided $\lim_{x\to 1^-} f(x)$ is different from the right-sided $\lim_{x\to 1^+} f(x)$, we conclude $\lim_{x\to 1} f(x)$ does not exist.

   From the information provided about function g, we observe that as x approaches 1 from the left, the value of g is close to 0, and as x approaches 1 from the right, the value of g is also close to 0. The table also indicates that $g(1)$ is undefined, but this information has no impact on the limit as x approaches 1. Since the table suggests that $\lim_{x\to 1^-} g(x) = \lim_{x\to 1^+} g(x) = 0$, we conclude $\lim_{x\to 1} g(x) = 0$.

   From the information provided about function h, we observe that as x approaches 1 from the left, the value of h is close to 0, and as x approaches 1 from the right, the value of h is also close to 0. The table also indicates that $h(1) = 1$, but this value has no impact on the limit as x approaches 1. Since the table suggests that $\lim_{x\to 1^-} h(x) = \lim_{x\to 1^+} h(x) = 0$, we conclude $\lim_{x\to 1} h(x) = 0$. The answer is D.

7. The notation $\lim_{x\to 2}(x^3+3x-4) = 10$ means that the value of the function $f(x) = x^3+3x-4$ can be made as close as we please to 10 by choosing x sufficiently close to, but not equal to, 2.

8. Create a table to evaluate $f(x) = \frac{e^x-1}{x}$ at values of x near 0, choosing numbers x slightly less than 0 and numbers x slightly greater than 0.

	numbers x slightly less than 0 →				← numbers x slightly more than 0		
x	-0.1	-0.01	-0.001	$\to 0 \leftarrow$	0.001	0.01	0.1
$f(x) = \frac{e^x-1}{x}$	0.9516	0.9950	0.9995	$f(x)$ approaches 1	1.0005	1.0050	0.0517

The table suggests that the value of $f(x) = \frac{e^x-1}{x}$ can be made as close as we please to 1 by choosing x sufficiently close to 0. This suggests that $\lim_{x\to 0} \frac{e^x-1}{x} = 1$.

1.2) Limits of Functions using Properties of Limits

10. $\boxed{\text{True}}$. If $f(x) = \dfrac{(x+1)(x+2)}{x+1}$ and $g(x) = x+2$, then $\lim_{x\to -1} f(x) = \lim_{x\to -1} g(x) = 1$.

36. Observe that the limit of the denominator is equal to zero. Factoring the denominator and simplifying yields

$$\lim_{x\to -2} \frac{x+2}{x^2-4} = \lim_{x\to -2} \frac{x+2}{(x-2)(x+2)} = \lim_{x\to -2} \frac{1}{x-2} = \boxed{-\frac{1}{4}}.$$

40. Observe that

$$\frac{3x}{x-2} - \frac{6}{x-2} = \frac{3x-6}{x-2} = \frac{3(x-2)}{x-2} = 3,$$

provided that $x \neq 2$. Therefore,

$$\lim_{x\to 2} \left(\frac{3x}{x-2} - \frac{6}{x-2} \right) = \lim_{x\to 2} 3 = \boxed{3}.$$

48. Observe that the limit of the denominator is equal to zero. Factoring the numerator and simplifying yields

$$\lim_{x\to 3^+} \frac{x^2-9}{x-3} = \lim_{x\to 3^+} \frac{(x-3)(x+3)}{x-3} = \lim_{x\to 3^+} (x+3) = 3+3 = \boxed{6}.$$

54. Because the limit of the denominator is not equal to zero,

$$\lim_{x\to c} \frac{f(x)}{g(x)-h(x)} = \frac{\lim_{x\to c} f(x)}{\lim_{x\to c}(g(x)-h(x))} = \frac{\lim_{x\to c} f(x)}{\lim_{x\to c} g(x) - \lim_{x\to c} h(x)} = \frac{5}{2-0} = \boxed{\frac{5}{2}}.$$

56. $\lim_{x\to c} [4f(x)\cdot g(x)] = 4 \lim_{x\to c} f(x) \cdot \lim_{x\to c} g(x) = 4(5)(2) = \boxed{40}.$