HW KEY (10/1 – 10/5)

2.1 Rates of Change and the Derivative (#6, 32, 51, 54)

6. derivative

32.

Time Interval	Start t_0	End t	Δt	$\frac{\Delta s}{\Delta t} = \frac{f(t)-f(t_0)}{t-t_0} = \frac{(5-t^2)-4}{t-1}$
$[1, 1.1]$	1	1.1	0.1	$\frac{\Delta s}{\Delta t} = \frac{f(1.1)-f(1)}{1.1-1} = \frac{(5-(1.1)^2)-4}{0.1} = -2.1$
$[1, 1.01]$	1	1.01	0.01	$\frac{\Delta s}{\Delta t} = \frac{f(1.01)-f(1)}{1.01-1} = \frac{(5-(1.01)^2)-4}{0.01} = -2.01$
$[1, 1.001]$	1	1.001	0.001	$\frac{\Delta s}{\Delta t} = \frac{f(1.001)-f(1)}{1.001-1} = \frac{(5-(1.001)^2)-4}{0.001} = -2.001$

The velocity appears to be approaching $\boxed{-2 \text{ cm/s}}$.

51. (a) The volume of the cube is given by $V = V(x) = x^3$. So the average rate of change of volume is $\frac{\Delta V}{\Delta x} = \frac{V(2.01)-V(2.00)}{2.01-2.00} = \frac{(2.01)^3-(2.00)^3}{0.01} = \frac{0.120601}{0.01} = \boxed{12.060 \text{ centimeters}^3/\text{centimeter}}$.

(b) $V'(2) = \lim_{x\to 2} \frac{V(x)-V(2)}{x-2} = \lim_{x\to 2} \frac{x^3-2^3}{x-2} = \lim_{x\to 2} \frac{(x-2)(x^2+2x+4)}{x-2} =$ $\lim_{x\to 2} (x^2 + 2x + 4) = \boxed{12 \text{ centimeters}^3/\text{centimeter}}$.

54. (a) The derivative, $d'(c)$, represents the rate of change of demand per cost per gallon of olive oil.

(b) $\boxed{d'(30) \text{ is larger than } d'(5)}$. We see that the slope of the tangent line at $c = 30$ is greater than the slope of the tangent line at $c = 5$, and we estimate that $d'(30)$ is

near zero, but $d'(5)$ is significantly less than zero. $d'(5)$ represents the rate of change of demand for olive oil (in gallons) per change in cost per gallon, in dollars, when the cost c is 5 dollars. Similarly, $d'(30)$ represents the rate of change of demand for olive oil (in gallons) per change in cost per gallon, in dollars, when the cost c is 30 dollars.

2.2 The Derivative as a Function (#46, 49, 58, 62, 65)

46. (a) Yes. Since the derivative $f'(x)$ equals the slope of a tangent line, horizontal tangent lines occur where the derivative equals 0. Since $f'(x) = 0$ for $x = -4$ and $x = 0$, the graph of f has two horizontal tangent lines, one at the point $(-4, f(-4))$ and the other at $(0, f(0))$.

(b) Yes. The graph of f has a vertical tangent line at $x = -2$ because both one-sided limits at $x = -2$ are infinite. Since one of the one sided limits equals $-\infty$ and the other equals ∞, f has a cusp at $(-2, f(-2))$.

(c) Yes. The graph of f has a corner at $x = 2$ because the graph of f' has unequal limits as x approaches 2. So the graph of f has a corner at $(2, f(2))$.

49. (a) At $\boxed{c = -2 \text{ and } c = 4}$, $\lim_{x \to c} f(x)$ exists, but f is not continuous.

(b) At $\boxed{c = 0,\ c = 2, \text{ and } c = 6}$, f is continuous, but f is not differentiable.

58. Let $\boxed{f(x) = x^3}$ and $\boxed{c = 2}$. Then $f'(2) = \lim_{h \to 0} \frac{f(2+h) - f(2)}{h} = \lim_{h \to 0} \frac{(2+h)^3 - 8}{h}$.

62. Let $\boxed{f(x) = \cos x}$ and $\boxed{c = \frac{\pi}{4}}$. Then $f'\left(\frac{\pi}{4}\right) = \lim_{x \to \frac{\pi}{4}} \frac{f(x) - f\left(\frac{\pi}{4}\right)}{x - \frac{\pi}{4}} = \lim_{x \to \frac{\pi}{4}} \frac{\cos x - \frac{\sqrt{2}}{2}}{x - \frac{\pi}{4}}$.

65. $V'(t) = \lim_{h \to 0} \frac{V(t+h) - V(t)}{h} = \lim_{h \to 0} \frac{4(t+h) - 4t}{h} = \lim_{h \to 0} \frac{4t + 4h - 4t}{h} = \lim_{h \to 0} \frac{4h}{h} = \lim_{h \to 0} 4 = \boxed{4}$.

The units of $V'(t)$ are cubic feet per second.